The University of Nottingham

SCHOOL OF PHYSICS & ASTRONOMY

A LEVEL 1 MODULE, AUTUMN SEMESTER 2004-2005

MECHANICS

Time allowed TWO Hours

Candidates must NOT start writing their answers until told to do so

Answer Section A and Section B

Only silent, self contained calculators with a Single-Line Display or Dual-Line Display are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

*An indication is given of the approximate weighting of each part of a question by means of a bold figure enclosed by curly brackets, e.g. {**2**}, immediately following that part.*

DO NOT turn examination paper over until instructed to do so

Speed of light in free space	c	$3.00\text{x}10^{8}$ m s^{-1}
Gravitational constant	G	$6.67\text{x}10^{-11}$ N m^2 kg^{-2}
Planck's constant	h	$6.63\text{x}10^{-34}$ J s
	$\hbar$	$1.055\text{x}10^{-34}$ J s
Elementary charge	e	$1.60\text{x}10^{-19}$ C
Mass of electron	m_e	$9.11\text{x}10^{-31}$ kg
Mass of proton	m_p	$1.6726\text{x}10^{-27}$ kg
Mass of neutron	m_n	$1.6749\text{x}10^{-27}$ kg
Boltzmann's constant	k_B	$1.38\text{x}10^{-23}$ J K^{-1}
Gas constant	R	8.31 J K^{-1} mol^{-1}
Permittivity of free space	ε_0	$8.85\text{x}10^{-12}$ F m^{-1}
Permeability of free space	μ_0	$4\pi\text{x}10^{-7}$ H m^{-1}
Bohr magneton	μ_B	$9.27\text{x}10^{-24}$ J T^{-1}
Stefan-Boltzmann constant	σ	$5.567\text{x}10^{-8}$ W m^{-2} K^{-4}
Avogadro's number	N_A	$6.02\text{x}10^{26}$ $kmol^{-1}$

Radius of Earth = 6400 km

Additional Material: A Sheet of Moments of Inertia will be provided

SECTION A

You may answer as many questions as you wish. Maximum credit of 15 marks will be awarded and any marks in excess of 15 will **not** be counted.
You should aim to spend about 40 minutes on this section.

1. A boy anchors his boat in the middle of a wide river in which a current of 2 m s^{-1} is flowing due South. The boy can swim at 3 m s^{-1} relative to the water. Calculate the minimum time required for him to swim to a buoy 100 m to the South of the boat and back again. {**2½**} What is the shortest time that it would take the boy to swim to a buoy 100 m to the West and back again? {**2½**}

2. Three small blocks A, B, and C are joined together by two light inextensible strings, one between A and B, and the other between B and C. The masses of the blocks are m_A = 3 kg, m_B = 5 kg, m_C = 4 kg respectively. The system is pulled along a flat, horizontal sheet of ice in a straight line by applying a steady force $\mathbf{F}$ to a third string tied to block A. The coefficient of kinetic friction between the blocks and the ice is μ_k = 0.2. If the system accelerates at 3 m s^{-2}, calculate the tension in each of the three strings. {**5**}

3. Consider a uniform, thin rectangular sheet of metal of mass m with width a and length b. Calculate the moment of inertia about

   (a) an axis that is in the plane of the sheet and passes through the sheet's centre of mass and parallel to the side of length b. {**2**}

   (b) an axis that is perpendicular to the plane of the sheet and passes through the sheet's centre of mass. {**1**}

   (c) an axis that is perpendicular to the plane of the sheet and passes through the sheet's corner. {**2**}

4. A force $\mathbf{F} = ky^2(\mathbf{i} + 3x^2\,\mathbf{j})$ acts in the x-y plane, where k is a constant. Referring to Fig. Q.4, evaluate the work done against this force when moving

   (a) O to A to B (*Path 1*), {**2**}

   (b) O to C to B (*Path 2*). {**2**}

   What statements can be made about whether the force is conservative or non-conservative? {**1**}

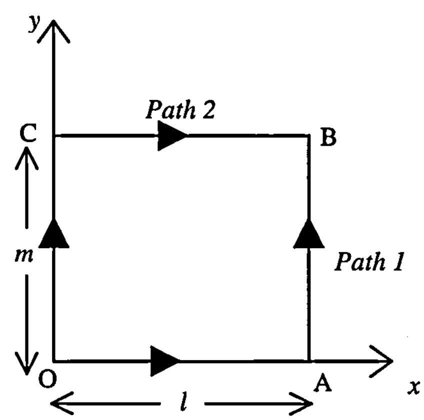


FIG. Q.4

5. A roundabout of diameter 3.4 m and moment of inertia 120 kg m^2 about its axis of rotation is pushed to an angular speed of 2.5 rad s^{-1}. A child of 50 kg steps on the edge of the roundabout. What is the new angular speed? **{5}**

SECTION B

Answer TWO questions only. Maximum credit of 30 marks will be awarded.
You should aim to spend about 80 minutes on this section.

6. Starting from Newton's second law of motion, show that the rocket equation can be expressed in vector notation as

$$\mathbf{F} = M\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} - \mathbf{u}\frac{\mathrm{d}M}{\mathrm{d}t}. \quad \{8\}$$

A stationary rocket of initial mass 10^3 kg is launched vertically from the surface of the Earth. The rocket expels mass at a rate of 120 kg s^{-1}, and the speed of the exhaust relative to the rocket is 10^4 m s^{-1}. What is the speed of the rocket 5 s after its launch? **{7}**

7. Show with the aid of a diagram the relation between plane polar (r, θ) and Cartesian co-ordinates (x, y) relative to the same origin. Write down algebraic expressions for x and y in terms of r and θ, and describe the plane polar unit vectors in terms of the Cartesian unit vectors. **{3}**

Given that we can describe the position in plane polar co-ordinates as $\mathbf{r} = r\hat{\mathbf{r}}$, show that we can express velocity, $\mathbf{v}$, as $\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$ and acceleration, $\mathbf{a}$, as $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}$, where all symbols have their usual meaning. **{6}**

A child stands on the edge of a roundabout of radius 10 m which is rotating at a steady rate of 4 revolutions/minute. At some time $t = 0$, the child starts running radially inwards towards the centre of the roundabout with a constant acceleration of 1 m s^{-2} (relative to the roundabout). Write down expresssions giving the velocity and acceleration of the child at a time t later, as seen by an observer on the ground. **{4}** Determine the magnitudes of the velocity and acceleration as seen by an observer on the ground when the child reaches the centre of the roundabout. **{2}**

8. (a) Define the terms *inertial frame* and *fictitious force*, illustrating your answer with a specific example of a 'mechanics' problem. {**5**}

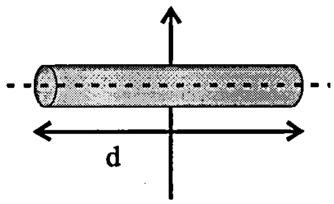
(b) Calculate the magnitude and direction of the Centrifugal and Coriolis forces at a latitude of 35°N on an aircraft of mass m = 60,000 kg flying East at 600 km h^{-1}. {**5**}

(c) In the absence of any perturbations, low pressure systems will always rotate one way in the Northern hemisphere and in the opposite direction in the Southern hemisphere. Discuss the origin of this observation, defining your frame of reference and stating any forces introduced. {**4**} For each hemisphere determine whether the rotation is clockwise or anticlockwise. {**1**}

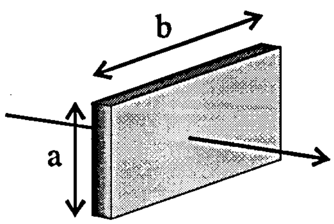
MECHANICS

Moments of Inertia

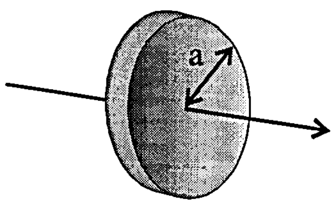
All objects solid of mass m and of uniform density.
All axes through centre of mass as shown.



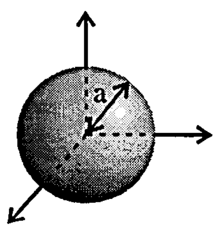
$$I = \frac{1}{12} m d^2$$



$$I = \frac{1}{12} m (a^2 + b^2)$$



$$I = \frac{1}{2} m a^2$$



Solid sphere: $I = \frac{2}{5} m a^2$

Hollow sphere: $I = \frac{2}{3} m a^2$