SR Problem with any Transformation other than Lorentz (or Poincare)

According to geometry along with any transformation of coordinates the Metric Tensor undergo a transformation as any second rank covariant tensor. Let us explain this on the example of Lorentz Metrics (LM), Lorentz Transformation (LT), and Galilean Transformation (GT). For simplification we consider only 2 dimensions $t=x^0$, $x=x^1$. Suppose we have original coordinate system S with the given Lorentz metrics:

$$g_{ik}=\begin{pmatrix}1,0\\0,-1\end{pmatrix} \qquad (1)$$

where the indexes i and k can be 0 or 1. Suppose we introducing another coordinate system S' by making an arbitrary transformation of coordinates:

$x'^k=x'^k(x^o,x^1)$ (2)

The partial derivatives of (2) are: dx'^i/dx^k (4 of them). Then we have the inverse transformation:

$x^k=x^k(x'^o,x'^1)$ (3)

The partial derivatives of (3) are: dx^i/dx'^k (4 of them).

The transformation law of a second rank covariant tensor (like the metric tensor (1)) is:

$$g'_{ik}=\frac{\partial x^a}{\partial x'^i}\frac{\partial x^b}{\partial x'^k}g_{ab} \qquad (4)$$

Here we have two contractions (indexes a and b). For our metric (1) (4) becomes:

$$g'_{ik}=\frac{\partial x^a}{\partial x'^i}\frac{\partial x^b}{\partial x'^k}g_{ab}=\frac{\partial x^0}{\partial x'^i}\frac{\partial x^0}{\partial x'^k}-\frac{\partial x^1}{\partial x'^i}\frac{\partial x^1}{\partial x'^k} \qquad (4a)$$

Instead of arbitrary transformation (2) let us consider a Lorentz Transformation (LT):

$x'^0=\gamma(x^0-vx^1)$, $x'^1=\gamma(x^1-vx^0)$ (2a)

and inverse: $x^0=\gamma(x'^0+vx'^1)$, $x^1=\gamma(x'^1+vx'^0)$ (3a)

Calculating (4a) for different indexes i and k and using LT (3a) we can prove that $g'_{ik}=g_{ik}$. This is a very important result saying that after LT the Lorentz Metric (1) conserves its exact form (the same components).

Let us now use not LT but Galilean Transformation (GT):

$$t'=t,\quad x'=x-vt,\quad \frac{\partial x'^i}{\partial x^k}=\begin{pmatrix}1,0\\-v,1\end{pmatrix} \qquad (2b)$$

then the inverse of that is:

$$t=t',\quad x=x'+vt',\quad \frac{\partial x^i}{\partial x'^k}=\begin{pmatrix}1,0\\v,1\end{pmatrix} \qquad (3b)$$

Calculating (4a) for different indexes i and k and using GT (3b) we get:

$$g'_{ik} = \begin{pmatrix} 1-v^2, -v \\ -v, -1 \end{pmatrix} \qquad (5)$$

What does it mean? It means that the physical time s of a clock that is at rest in S' is different from the value of time coordinate t':

$$s = t'\sqrt{g'_{00}} = t'\sqrt{1-v^2} \qquad (6).$$

The root we can call a "time root factor" (in general we can get a root factor for every coordinate axis). We can not do "measurements" in Galilean coordinates without making corrections on the time factor. Any calculation in S' will become more complicated and we rather avoid Galilean Transformation in SR.