The Milliken Moment Method - Part 1, By Terry Duell

The Milliken Moment Method (MMM) is a well developed approach to deriving a complete description of a vehicle's steady state handling characteristics.

Vehicles can be assumed to be steady state devices when it comes to correcting almost all handling problems. Fixes are almost always derived from steady state considerations.

Reference 1 describes the approach in detail. I would only recommend it for serious study of the subject. Ref. 2 would be a better source for additional information for most readers.

The MMM is one where the vehicle is operated under steady state conditions, on a constant radius turn, or at constant speed, and the lateral force and yaw moment (tends to rotate the vehicle about a vertical axis) are established for the range of possible steer and body slip angles. The results provide a complete 'map' of the manoeuvring and control ability of the vehicle.

This article will provide some basics, and following articles will go into more detail.

Curvilinear motion

When a vehicle travels around a constant radius turn at constant speed (fig. 1), the direction of motion is continuously changing, which means that the vehicle is accelerating laterally, and that a force is required to generate that lateral acceleration.

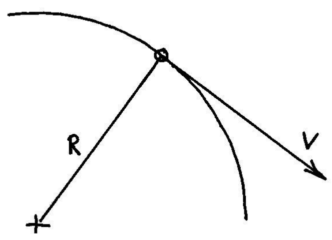

Figure 1. Curvilinear motion

The lateral acceleration, which is normal (at right angles) to the instantaneous velocity, and directed to the centre of the arc, is

$$a_n = \frac{V^2}{R}$$

where $V = velocity$, and $R = radius$

The force required is

$$F_n = \frac{Wa_n}{g} = \frac{WV^2}{Rg}$$

$$W = vehicle\ weight$$

and is directed toward the centre of the arc.

The force F_n must be generated by the tyres, although some small contributions may also come from gravity, if the turn has a side slope towards the centre of the turn, and from a cross wind.

Tyres

The tyres are the main source of the lateral, or side force that allows a vehicle to manoeuvre. Tyres generate lateral force when operated at a slip angle, and the magnitude of the lateral force depends on the normal load on the tyre (weight or mass), the tyre pressure, camber angle and the slip angle. Typical characteristic curves for a tyre, at varying normal load are shown in figure 2.

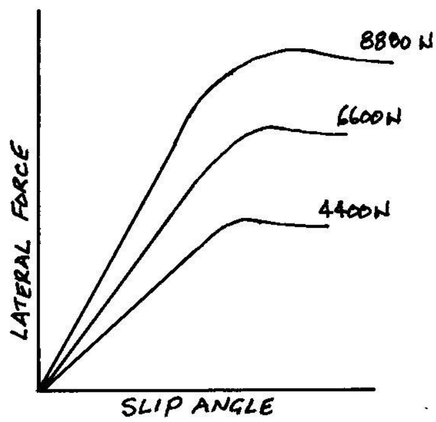

Figure 2. Tyre characteristics

The steady state turn

Now let's look at a generalised view of a vehicle on a constant radius turn.

Fig. 3 shows the steer angle (δ), the body sideslip angle (β), and the tyre slip angles (α). The tyre slip angles are the angle between the plane of the tyre and the direction of motion of the tyre.

Note that for a small radius of turn (R) the tyre slip angles would be different for left

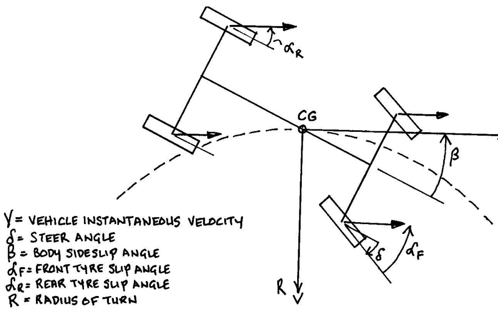

Figure 3. Steady state turn

and right, but for large radii relative to the wheelbase we can assume them to be equal. The steer angle may differ left to right as a result of steering geometry and initial settings (toe-in or toe-out), hence the front tyre slip angles may be different, left to right, regardless of the radius of turn.

If we know the velocity (V) and the radius (R), then we know the lateral acceleration (a_n) and the lateral force (F_n).

A lateral acceleration acting on the vehicle centre of gravity (CG) will cause the vehicle to roll on it' s suspension and there will be load transfer from the inside to outside tyres. The amount of load transfer will be determined by the suspension geometry and roll stiffnesses.

The roll stiffness at each axle is affected by the suspension springs, suspension geometry, and any auxiliary roll stiffness installed (e.g. roll or sway bars).

All these considerations will allow the lateral force at each tyre to be established, and it will act normal to the plane of the tyre, as shown in fig. 4.

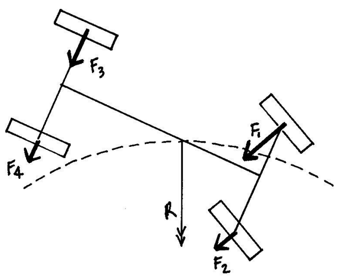

Figure 4. Tyre forces

In addition to the lateral tyre forces, there may also be driving or braking forces being applied, which have an effect on the ability of the vehicle to manoeuvre. In fact the vehicle may be accelerating or decelerating, and steady state conditions can still be considered to apply. The only requirement is that the forces acting on the vehicle remain constant.

The driving forces, which may be at front, rear or all tyres, will act in the plane of the tyre, as will any braking forces.

The net effect of all these forces is that there be a lateral force acting on the CG towards the centre of the arc, and possibly a yaw moment tending to rotate the vehicle either clockwise or anti-clockwise, as per fig. 5.

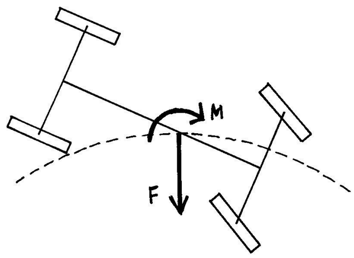

Figure 5. Lateral force and yaw moment

If the yaw moment is zero, the vehicle is said to be ' trimmed' . If the yaw moment i clockwise (positive) the vehicle will tend to rotate into the turn and is oversteering. If the yaw moment is negative, the vehicle will tend to rotate away from the turn and is understeering.

The full set of lateral force and yaw moments that the vehicle is capable of developing over the range of steer angle (δ) and body slip angles (β) make up the force-moment diagram which is the result of the MMM. The next article will describe these diagrams in more detail.

References

1. William F. Milliken Jr., Fred Dell' Amico, Roy S. Rice. "The Static Directional Stability and Control of the Automobile". SAE 760712, October 1976. (technical paper)

2. William F. Milliken, Douglas L. Milliken. " Race Car Vehicle Dynamics". SAE international 1995. (book)

[Terry Duell may be contacted at tduell@iinet.net.au]