we have

$$\sqrt{\tfrac{3}{5}}g_{\mathrm{SU}(5)} = g'. \tag{5.4.6}$$

But

$$g_{\mathrm{SU}(5)} = g. \tag{5.4.7}$$

It therefore follows that $\tan\theta_W = \sqrt{\frac{3}{5}}$, leading to the famous prediction $\sin^2\theta_W = \frac{3}{8}$. Incidentally, the same result is also obtained if we use eq. (5.3.4).

Next we would like to present the breakdown of the SU(5) group to $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{cm}}$. Within the conventional Higgs picture this is achieved by introducing two Higgs multiplets: one belonging to the {24}-dimensional irreducible representation (denoted by Φ), and the other to the {5}-dimensional (denoted by H) representations. The stages of symmetry breakdown are given by

$$\mathrm{SU}(5) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1) \to \mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}.$$

The first stage is achieved by $\langle\Phi\rangle \neq 0$ and the second stage by $\langle H\rangle \neq 0$ as follows: from the general group theory of spontaneous breakdown we can show that [5]

$$\langle\Phi\rangle = V\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ & & & & -3/2 \end{pmatrix}, \tag{5.4.8}$$

i.e., the breaking is along the λ_{24} direction. This is responsible for the first stage of the symmetry breakdown. The second stage is caused by

$$\langle H\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \rho/\sqrt{2} \end{pmatrix}. \tag{5.4.9}$$

In the presence of both H and Φ, the v.e.v. of Φ changes somewhat and looks like the following:

$$\langle\Phi\rangle = V\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2-\varepsilon/2 & \\ & & & & -3/2+\varepsilon/2 \end{pmatrix}. \tag{5.4.10}$$

It is then a straightforward matter to compute all the gauge boson masses

$$\text{Superheavy bosons } X, Y: \quad m_X^2 \approx m_Y^2 = \tfrac{25}{8}g^2V^2, \tag{5.4.11}$$