

$$\pi^\nu = T^{\cdot\nu}$$

$$\pi^0 = : \dot{\psi}^\dagger \dot{\psi} + (\vec{\nabla} \psi^\dagger) \cdot (\vec{\nabla} \psi) + m^2 \psi^\dagger \psi :$$

$$\vec{\pi} = - : \dot{\psi}^\dagger \vec{\nabla} \psi : + h.c.$$

The normal ordering is simplifying and necessary anyway. You can write the  $\hat{a}$  and  $\hat{b}$  and their h.c. operators in arbitrary order under the normal-ordering symbol.

Then you set

$$\vec{P}_1 = - \int d^3 \vec{x} : \dot{\psi} \vec{\nabla} \psi : \quad ; \quad \vec{P} = \vec{P}_1 + \vec{P}_1^\dagger$$

Now with the mode decomposition you set after some simple algebra  $\int d^3 \vec{p}_1$  by def.

$$\vec{P}_1 = - \int d^3 \vec{x} \int d^3 \vec{p}_1 \int d^3 \vec{p}_2 E_1 \vec{p}_2$$

$$: [i a_1^\dagger \exp i \vec{p}_1 \cdot \vec{x} - i b \exp -i \vec{p}_1 \cdot \vec{x}] [i a_2 \exp i \vec{p}_2 \cdot \vec{x} - i b_2^\dagger \exp i \vec{p}_2 \cdot \vec{x}] :$$

with  $a_n = a(\vec{p}_n)$  etc. and  $\exp_{\pm 1} = \exp(\pm i \vec{p}_1 \cdot \vec{x})$   $p_1^0 = E_1$   
etc. Then multiply out

$$\vec{P}_1 = + \int d^3 \vec{p}_1 \int d^3 \vec{p}_2 \int d^3 \vec{x} E_1 \vec{p}_2$$

$$: [a_1^\dagger a_2 \exp_{1+2} - b_1 a_2 \exp_{-1-2} - a_1^\dagger b_2^\dagger \exp_{1+2} + b_2^\dagger b_1 \exp_{-1+2}]$$

$$= \int d^3 \vec{p}_1 \frac{(2\pi)^3 E_1 \vec{p}_1}{(2\pi)^3 2E_1} [a_1^\dagger a_1 + b_1 a_1 + a_1^\dagger b_1^\dagger \exp_{1+2} + b_1^\dagger b_1]$$



$$\Rightarrow \vec{P}_1 = i \vec{P}_1 + i \vec{P}_1^\dagger$$

$$= \int d^3 \vec{p}_1 \frac{\vec{p}_1}{2} [2 a_1^\dagger a_1 + 2 b_1^\dagger b_1]$$

$$+ \int d^3 \vec{p}_1 \frac{\vec{p}_1}{2} [(b_1 a_{-1} + b_{-1} a_1 / \exp(-2i E_1 t)) + (b_1^\dagger a_{-1}^\dagger + b_{-1}^\dagger a_1^\dagger) \exp(+2i E_1 t)]$$

In the 2nd line you substitute  $\vec{p}_1 \rightarrow -\vec{p}_1$  in the 2nd terms in front of the exp fcts. Since  $E_1 = +E_{-1}$  This shows that this term drops, and you finally get

$$\vec{P} = \int d^3 \vec{p}_1 \vec{p}_1 (a_1^\dagger a_1 + b_1^\dagger b_1)$$

Note that your normalization of the mode function is such that

$$N_a(\vec{p}) = \frac{a^\dagger(\vec{p}) a(\vec{p})}{(2\pi)^3 2E(\vec{p})} \text{ and analogous for } b$$

The energy is ~~the~~ similar. It's clever to use the kb eq. to rewrite

$$P^0 = H = \int d^3 \vec{x} : [\dot{\psi}^\dagger \dot{\psi} - \psi^\dagger \psi'] :$$

The rest is similar to the calculation for  $\vec{P}$ .