



Tutorial on Basic Motion of an Object

Tools Required

- A bit of calculus (e.g. integration and differentiation)
- A little algebra (e.g. solving quadratic equations, 2 equations w/ 2 unknowns)
- Some geometry and trigonometry (e.g. angles, sine, cosine, tangent, etc.)
- An open mind willing to learn

Premise

- Physics of motion can be very confusing if you have equations provided to you and are not sure which ones apply nor how to use them to solve the problem at hand.
- Most if not all elementary motion problems (e.g. free falling body, projectiles, incline planes) can be solved by the following simple methodology if followed properly:
 - 1) Choose a convenient coordinate system
 - 2) Draw a free body diagram (FBD) depicting the object of interest and the forces acting on it
 - 3) Write $\sum F = ma$ in each coordinate direction of interest
 - 4) Integrate the equation(s) derived in (3) once to get velocity, and again to get distance
 - 5) Apply known initial conditions from the problem statement (e.g. initial velocity, acceleration, distance) to the equations from (4)
 - 6) Use the equations from (5) with information in the problem statement to get what you want. Look for clues in the problem statement



Newton's Second Law

“The **change in velocity** (acceleration) with which an object moves is directly proportional to the magnitude of the force applied to the object and inversely proportional to the mass of the object.”

Mathematically, this can be stated: $\vec{F} = m\vec{a}$ where F and a are vector quantities. For analyzing problems, it can be convenient to write these equations for each coordinate direction (only cartesian is given here):

$$\Sigma F_x = ma_x \text{ or } \Sigma F_x = m \frac{dv_x}{dt} \text{ or } \Sigma F_x = m \frac{d^2x}{dt^2} \text{ or } \Sigma F_x = m \ddot{x} \quad (a_x = dv_x/dt, \quad v_x = dx/dt)$$

$$\Sigma F_y = ma_y \text{ or } \Sigma F_y = m \frac{dv_y}{dt} \text{ or } \Sigma F_y = m \frac{d^2y}{dt^2} \text{ or } \Sigma F_y = m \ddot{y} \quad (a_y = dv_y/dt, \quad v_y = dy/dt)$$

So, if we know the force acting on an object and its mass, we can integrate to obtain velocity and integrate again to find distance. So, the degree of difficulty with integrating the above depends on the complexity of **F**. Let's take a simple example where **F** equals mass under the influence of gravity, or **mg**:

$$m g = m \frac{dv_y}{dt}$$

Separating the variables, dividing both sides by m, and integrating, we get:

$$\int g dt = \int dv_y \iff gt + C_1 = v_y + C_2 \iff v_y = gt + C_1 - C_2$$

Where C_1 and C_2 are the arbitrary integration constants we learned to love in Calculus. Since they are arbitrary, there is no reason we can't combine them into a single arbitrary integration constant and call it v_{y0} , yielding:

$$\boxed{v_y = g t + v_{y0}} \quad (1.)$$

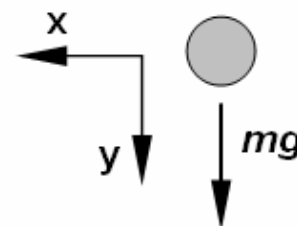
Eureka moment: did you ever wonder why in calculus the integration constants were important? In this case, it represents the initial velocity in the y direction. Why? Hint: what is v_y when $t = 0$?

Similarly, to get **y**, we just integrate the equation for v_y and combine integration constants:

$$v_y = g t + v_{y0} = dy/dt \iff \int dy = \int (gt + v_{y0}) dt$$

$$\boxed{y = \frac{1}{2}gt^2 + v_{y0}t + y_0} \quad (2.) \quad y_0 = \text{initial displacement at } t = 0$$

I. Free falling body – of mass m , initially at rest.



1. Coordinate system and FBD

2. Equation of motion in y

$$\sum F_y = ma_y:$$

Integrate a_y to get v_y :

Integrate v_y to get y :

$$mg = ma_y \rightarrow g = a_y$$

$$v_y = gt + v_{y0}$$

$$y = \frac{1}{2}gt^2 + v_{y0}t + y_0$$

3. Equation of motion in x :

$$\sum F_x = ma_x:$$

Integrate a_x to get v_x :

Integrate v_x to get x :

$$0 = a_x$$

$$v_x = v_{x0}$$

$$x = v_{x0}t + x_0$$

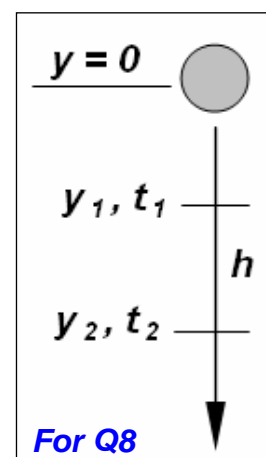
4. Apply initial conditions: $v_{x0} = v_{y0} = x_0 = y_0 = 0$

$$v_x = 0$$

$$v_y = gt$$

$$x = 0$$

$$y = \frac{1}{2}gt^2$$



Q1: why is a_y positive? Coordinate system choice (positive y is down).

Q2: why are v_x and $x = 0$? No force or initial velocity in x direction.

Q3: why are x_0 and y_0 equal 0? Clever choice of coordinate system.

Q4: why are v_{x0} and $v_{y0} = 0$? Problem statement: 'initially at rest'

Q5: what is the velocity at $t = t_1$? Substitute t_1 into velocity equation to find v .

Q6: what distance does the object fall in time $= t_1$? Substitute t_1 into equation for y .

Q7: How long does it take for the object to fall a distance h ? Set $y = h$ and solve for t (Hint: sqrt is involved).

Q8: At some point, the object falls a distance h in s seconds. What was the total distance fallen prior to this event? We need y_1 (see picture above). Using the expression for y , we can write $y_1 = \frac{1}{2}gt_1^2$ and $y_2 = \frac{1}{2}gt_2^2$. From the problem statement, we know that $y_2 - y_1 = h$ and $t_2 - t_1 = s$. These are 2 equations in 2 unknowns that can be solved by substitution for t_1 and y_1 .