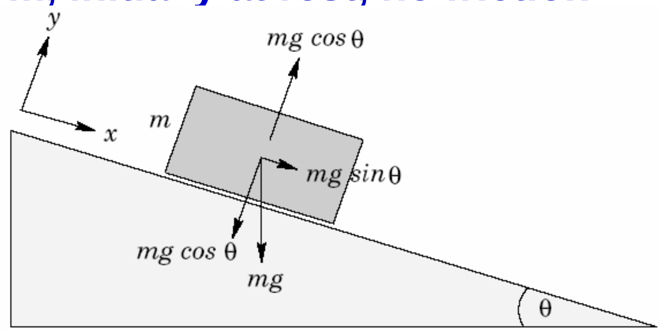
II. Body on incline – mass m, initially at rest, no friction



1. **Coordinate system and FBD**

2. **Equation of motion in *y***

   $\sum F_y = ma_y$: $0 = ma_y \rightarrow 0 = a_y$

   Integrate a_y to get v_y: $v_y = v_{yo}$

   Integrate v_y to get y: $y = v_{yo}t + y_o$

3. **Equation of motion in *x*:**

   $\sum F_x = ma_x$: $g\sin(\theta) = a_x$

   Integrate a_x to get v_x: $v_x = g\sin(\theta)t + v_{xo}$

   Integrate v_x to get x: $x = \frac{1}{2} g\sin(\theta)t^2 + v_{xo}t + x_0$

4. **Apply initial conditions:** $v_{x0} = v_{y0} = x_0 = y_0 = 0$

   $v_x = g\sin(\theta)t$ $\quad$ $x = \frac{1}{2} g\sin(\theta)t^2$

   $v_y = 0$ $\quad$ $y = 0$

Q1: why is a_x positive? It's pointing down! Coordinate system choice (positive x is down and to the right along the incline).

Q2: why are v_y and y = 0? The block remains on the incline and doesn't lift off. Gravity keeps in contacting the incline. Look at the direction of y.

Q3: why are x_0 and y_0 equal 0? Clever choice of coordinate system.

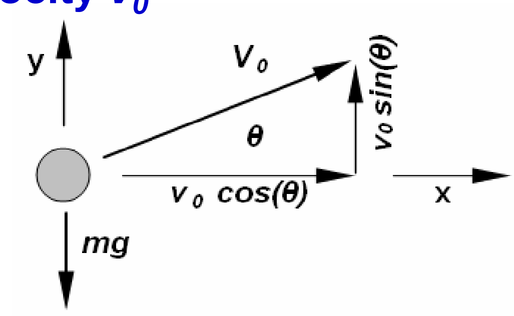
Q4: why are v_{x0} and v_{x0} = 0? Problem statement:: 'initially at rest'

Q5: how much time is required for the block to slide a distance h? Set $x = h$ and solve for t (hint: sqrt is involved)

Q6: what happens if θ increases and reaches 90^0? Same as free falling body.

Q7: what effect does friction have? Introduces a force in the negative x direction with magnitude μN, where N is equal and opposite to the component of mg in the y direction. More complicated F term in the equation of motion in x. Slightly more complicated equation for v_x and x.

III. Projectile – mass m, initial velocity v_0



1. **Coordinate system and FBD**

2. **Equation of motion in *y***

   $\sum F_y = ma_y$: $-mg = ma_y \rightarrow -g = a_y$

   Integrate a_y to get v_y: $v_y = -gt + v_{yo}$

   Integrate v_y to get y: $y = -\frac{1}{2} gt^2 + v_{yo}t + y_o$

3. **Equation of motion in *x*:**

   $\sum F_x = ma_x$: $0 = ma_x \rightarrow 0 = a_x$

   Integrate a_x to get v_x: $v_x = v_{xo}$

   Integrate v_x to get x: $x = v_{x0}t + x_0$

4. **Apply initial conditions:** $v_{x0} = v_0\cos(\theta)$, $v_{y0} = v_0\sin(\theta)$, $x_0 = y_0 = 0$

   $v_x = v_0\cos(\theta)$ $x = v_0\cos(\theta)t$

   $v_y = -gt + v_0\sin(\theta)$ $y = -\frac{1}{2} gt^2 + v_o\sin(\theta)t$

Q1: why is a_y negative? Coordinate system choice (positive is up).

Q2: why is v_x a constant? Derivative of a const = 0. Acceleration is the derivative of velocity and we know $a_x = 0$, so v_x must be constant.

Q3: why do x_0 and y_0 equal 0? Clever choice of coordinate system.

Q4: what is the time, t_f, when the trajectory is finished? Hint: what can we say about y at the end? Set $y = 0$ and solve for t (hint: sqrt is involved)

Q5: what is the range of the object? Take t_f from Q4 and calculate x.

Q6: what is the max height of the object? Hint: max/min ➔ derivative. Take the derivative of y, set equal to zero, solve for t, and use t to find y. But, we already have the derivative of y – it's velocity. So, set $v_y = 0$, solve for t, then calculate y. Either way works.

Q7: will the object clear a fence h height and d distance away? Set $x = d$ and find t, use the t to find y. If $y > h$, the answer is yes.

Epilog

1) If the previous three examples are studied carefully, it should be readily seen that the method in each case is identical. The only difference is how complicated F is (and the resulting integration).
2) By far the most challenging part of the physics of elementary motion is the proper construction of the Free Body Diagram (FBD) to get the forces acting on the body. To be sure, it can be tricky when the problems involve friction, pulleys with masses on table tops, etc. Statics are needed to determine the tension on ropes and normal forces on objects subject to friction forces. One comment about ropes and cables. The tension T is always the same on both ends. Also, it can be confusing if the problem statement says there is no force on the object but it is subject to a constant acceleration or velocity. In these cases, FDB's may not be needed because the equation of motion is already provided.
3) Once a proper FBD is constructed, writing the equations of motion and integrating to get velocity and distance is incredibly easy. The integrals are likely to be among the simplest ones ever encountered.
4) After the general equations for velocity and distance are derived, evaluating the integration constants is normally very straightforward using the initial conditions that are given in the problem statement. Look for clues like 'at rest', 'initial velocity = xxx', etc.
5) Once the initial conditions have been applied yielding the completed equations of motion (***for the specific problem***), answering the questions is usually straightforward, but can be tricky. The clues are in the problem statement. The key is to write them in mathematical terms. Some examples:
   - 'find the rate of change of …' means differentiation is likely
   - 'falls a distance h in r seconds' likely means there will be 2 equations in y, where $h = y_2 - y_1$ at t_2 and t_1, and $t_2 – t_1 = r$.
   - Etc.