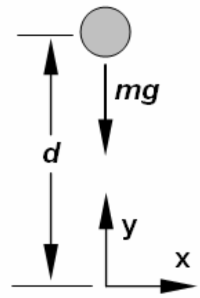
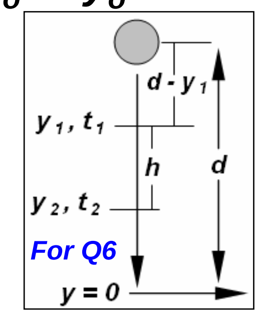
II. Free falling body - mass m, initially at rest at height d



1. **FBD and coordinate system**
   Choose +*y* up (opposite direction as *g*)

2. **Equation of motion in *y***
   $\sum Fy = ma_y$ (*g* in -*y* direction): $(-mg) = ma_y \rightarrow -g = a_y$
   Integrate a_y to get v_y: $v_y = -gt + v_{yo}$
   Integrate v_y to get *y*: $y = -\frac{1}{2}gt^2 + v_{yo}t + y_o$



3. **Equation of motion in *x*:**
   $\sum F_x = ma_x$ (no force in *x* direction): $0 = a_x$
   Integrate a_x to get v_x: $v_x = v_{xo}$
   Integrate v_x to get *x*: $x = v_{xo}t + x_0$

4. **Apply initial conditions (see Q2):** $v_{x0} = v_{y0} = x_0 = 0$, $y_0 = d$
   $v_x = 0$ $\quad$ $x = 0$
   $v_y = -gt$ $\quad$ $y = -\frac{1}{2}gt^2 + d$

Q1: Why are a_y and v_y negative? Coordinate system choice (positive *y* is up, *g* is down, motion is down).

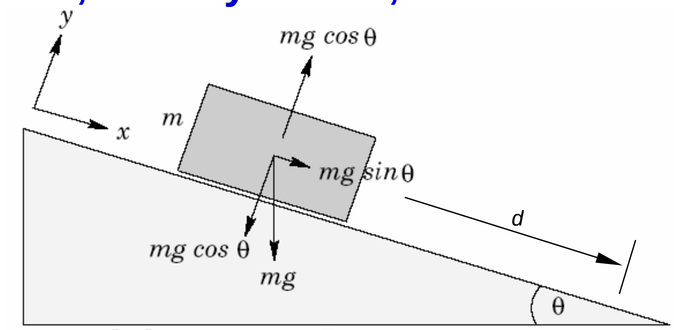
Q2: Why are v_x and $x = 0$? No force or initial velocity in *x* direction. Why is $x_0 = 0$? Coordinate origin at t = 0 position for x. Why is $y_0 = d$? Problem statement. Why are v_{x0} and $v_{x0} = 0$? Problem statement: 'initially at rest'

Q3: What is the velocity at $t = s_1$? Substitute $t = s_1$ into velocity equation to find *v* (Ans. $-gs_1$). What distance does the object fall in time = s_1? Need $d - y_{s1}$. Substitute $t = s_1$ into equation for *y* and subtract result from *d*.

Q5: How long does it take for the object to fall distance d? Set $y = 0$ (Hint: object starts out at $y = d$) and solve for *t*. Hint: square root is involved..

Q6: At some point, the object falls a distance *h* in *s* seconds. What was the distance fallen prior to this event? We need $d - y_1$ (see picture above). Using the expression for *y*, write equations for y_1 and y_2. From the problem statement, we know that $y_2 - y_1 = h$ and $t_2 - t_1 = s$. These are 2 equations in 2 unknowns that can be solved by substitution for t_1 and y_1.

III. Body on incline – mass m, initially at rest, no friction



1. **FBD and coordinate system**
   Choose +x along the incline

2. **Equation of motion in *y***
   $\sum Fy = ma_y$ (F_y = 0, see Q2): $(0) = ma_y \Rightarrow 0 = a_y$
   Integrate a_y to get v_y: $v_y = v_{yo}$
   Integrate v_y to get y: $y = v_{yo}t + y_o$

3. **Equation of motion in *x* (m on both sides of equation cancel):**
   $\sum F_x = ma_x$ (see Q1): $(mg \sin\theta) = ma_x$
   Integrate a_x to get v_x: $v_x = g \sin(\theta)t + v_{xo}$
   Integrate v_x to get x: $x = ½\, g \sin(\theta)t^2 + v_{xo}t + x_0$

4. **Apply initial conditions: $v_{x0} = v_{y0} = x_0 = y_0 = 0$ (see Q2, Q3)**
   $v_x = g \sin(\theta)t$ $x = ½\, g \sin(\theta)t^2$
   $v_y = 0$ $y = 0$

Q1: why is a_x positive? Coordinate system choice (positive x is along the incline, x component of mg is in positive direction).

Q2: why are v_y and y = 0? The block stays on the incline due to gravity. The incline imparts an equal an opposite reaction force to y component of mg. Why is a_y = 0. Forces in y direction cancel each other out; net F is zero.

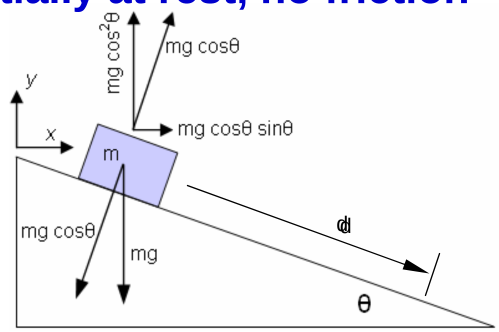
Q3: why are x_0 and y_0 equal 0? Choose coordinate origin at t = 0 position. Why are v_{x0} and v_{x0} = 0? Problem statement: 'initially at rest'

Q4: how much time is required for the block to slide a distance d? Set $x = d$ and solve for t (Hint: square root is involved)

Q5: what happens if θ increases and reaches 90^0? Same as free falling body.

Q6: what effect does friction have? Introduces a force in the $-x$ direction with magnitude μN, where N is equal and opposite to the component of mg in the y direction and μ is friction coefficient. More complicated F in the equation of motion in x. Slightly more complicated equation for v_x and x.

IV. Body on incline – mass m, initially at rest, no friction



1. **FBD and coordinate system**
   Choose +x horizontal to the right

2. **Equation of motion in *y***
   $\sum F_y = ma_y$: $(mg\cos^2\theta - mg) = ma_y$
   Integrate a_y to get v_y (see Q1): $v_y = -g\sin^2(\theta)t + v_{yo}$
   Integrate v_y to get y: $y = -\frac{1}{2} g\sin^2(\theta)t^2 + v_{yo}t + y_o$

3. **Equation of motion in *x*:**
   $\sum F_x = ma_x$: $(mg\sin\theta\cos\theta) = ma_x$
   Integrate a_x to get v_x: $v_x = g\sin(\theta)\cos(\theta)t + v_{xo}$
   Integrate v_x to get x: $x = \frac{1}{2} g\sin(\theta)\cos(\theta)t^2 + v_{xo}t + x_0$

4. **Apply initial conditions: $v_{x0} = v_{y0} = x_0 = y_0 = 0$ (see Q2)**
   $v_x = g\sin(\theta)\cos(\theta)t$ $x = \frac{1}{2} g\sin(\theta)\cos(\theta)t^2$
   $v_y = -g\sin^2(\theta)t$ $y = -\frac{1}{2} g\sin^2(\theta)t^2$

Q1: Why does $mg\cos^2\theta - mg$ integrate to $-g\sin^2(\theta)t + v_{yo}$? Factor out mg and cancel m from both sides, we get $g(\cos^2\theta - 1) = a_y$. Using the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ we get $\cos^2\theta - 1 = -\sin^2\theta$.

Q2: Why are x_0 and y_0 equal 0? Choose coordinate origin at t = 0 position. Why are v_{x0} and v_{x0} = 0? Problem statement: 'initially at rest'

Q3: How much time is required for the block to slide a distance d? Using the geometry of the incline, we get $x^2 + y^2 = d^2$ (i.e. Pythagorean theorem). Substitute the expressions for x and y into this equation and solve for t.

Comment: This example illustrates the importance of choosing an appropriate coordinate system. By choosing x to be horizontal rather than along the incline plane, the result is equations of motion in 2 directions instead of one and a much more complex solution to Q3, which yields the exact same answer as in problem III. Verification of this is left as a exercise for the student.