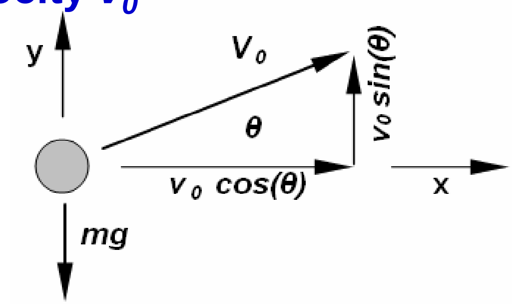
V. Projectile – mass m, initial velocity v_0



1. **FBD and coordinate system**
   Choose +*y* up (g is down)

2. **Equation of motion in *y*:**
   $\sum Fy = ma_y$ (g is negative, see Q1): $(-mg) = ma_y \rightarrow -g = a_y$
   Integrate a_y to get v_y: $v_y = -gt + v_{yo}$
   Integrate v_y to get y: $y = -\frac{1}{2} gt^2 + v_{yo}t + y_o$

3. **Equation of motion in *x* (no force in x direction):**
   $\sum F_x = ma_x$ (no x force, see Q2): $(0) = ma_x \rightarrow 0 = a_x$
   Integrate a_x to get v_x: $v_x = v_{xo}$
   Integrate v_x to get x: $x = v_{x0}t + x_0$

4. **Apply initial conditions:** $v_{x0} = v_0\cos(\theta)$, $v_{y0} = v_0\sin(\theta)$, $x_0 = y_0 = 0$
   $v_x = v_0\cos(\theta)$ $\qquad$ $x = v_0\cos(\theta)t$
   $v_y = -gt + v_0\sin(\theta)$ $\qquad$ $y = -\frac{1}{2} gt^2 + v_o\sin(\theta)t$

Q1: why is g negative? Coordinate system choice (*y* is positive up, gravity acts down)

Q2: Why is $a_x = 0$? No force in *x* direction, therefore $a_x = 0$. Why is v_x a constant? Must be constant because acceleration is the derivative of velocity and the derivative of a const = 0.

Q3: why do x_0 and y_0 equal 0? Choose coordinate origin at t = 0 position.

Q4: what is the time, t_f, when the trajectory is finished? Hint: what can we say about *y* at the end? Set *y* = 0 and solve for *t* (hint: square root is involved)

Q5: what is the range of the object? Take t_f from Q4 and calculate *x*.

Q6: what is the max height of the object? Hint: max/min → derivative. Take the derivative of *y*, set equal to zero, solve for *t*, and use *t* to find *y*. But, we already have the derivative of *y* – it's velocity. So, set $v_y = 0$, solve for *t*, then calculate *y*. Either way works.

Q7: will the object clear a fence *h* height and *d* distance away? Set *x* = *d* and find *t*, use the *t* to find *y*. If $y > h$, the answer is yes.

Epilogue

1) If the previous five examples are studied carefully, it should be readily apparent that the method in each case is identical. The only difference is how complicated F is (and the resulting integration).

2) By far the most challenging part of the physics of elementary motion is the proper construction of the Free Body Diagram (FBD) to get the forces acting on the body. To be sure, it can be tricky when the problems involve friction, pulleys with masses on table tops, etc. Statics are needed to determine the tension on ropes and normal forces on objects subject to friction forces. One comment about ropes and cables. The tension T is always the same on both ends. Also, it can be confusing if the problem statement says there is no force on the object but it is subject to a constant acceleration or velocity. In these cases, the equations of motion are easier because they are basically given to you.

3) Once a proper FBD is constructed, choosing the appropriate coordinate system can greatly simplify the problem. Choosing the origin of the coordinate system to coincide with the initial position of the object will make $x_0 = y_0 = 0$. Placing x along an incline reduces a sliding block on incline problem to 1 dimension.

4) After constructing the FBD and choosing the coordinate system, writing the equations of motion and integrating to get velocity and distance is incredibly easy. The integrals are likely to be among the simplest ones ever encountered.

5) After the general equations for velocity and distance are derived, evaluating the integration constants is normally very straightforward using the initial conditions that are given in the problem statement. Look for clues like ‘at rest’, ‘initial velocity = xxx’, etc.

6) Once the initial conditions have been applied yielding the completed equations of motion (***for the specific problem***), answering the questions is usually straightforward, but can be tricky. The clues are in the problem statement. The key is to write them in mathematical terms. Some examples:

   - ‘find the rate of change of …’ means differentiation is likely
   - ‘falls a distance h in r seconds’ likely means there will be 2 equations in y, where $h = y_2 - y_1$ at t_2 and t_1, and $t_2 - t_1 = r$.
   - Etc.