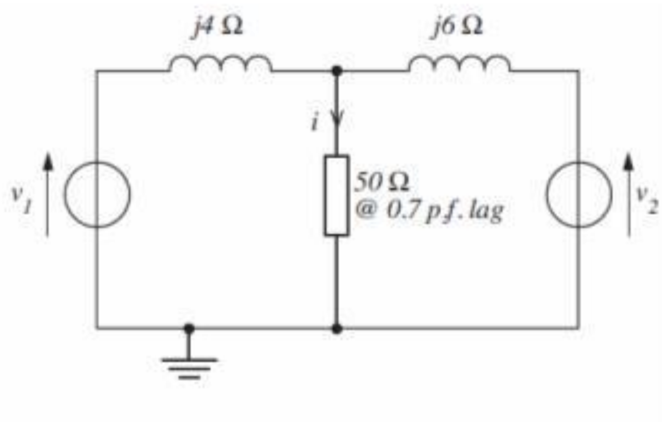
The diagram below shows a 50 Ω load being fed from two voltage sources via their associated reactances. Determine the current *i* flowing in the load by using:

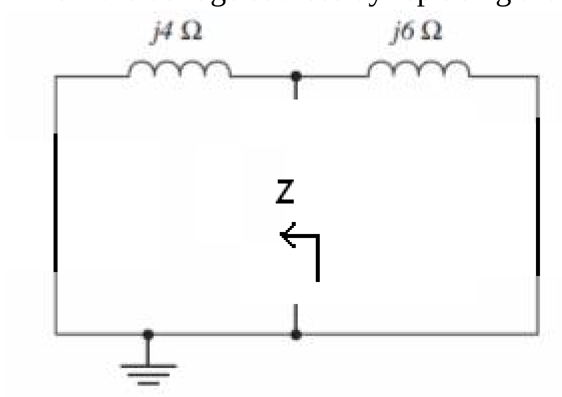
a) Thevenin's theorem
b) The superposition theorem



$$v_1 = \sqrt{2} \times 415 \cos(100\pi t) \text{ volts}$$
$$v_2 = \sqrt{2} \times 415 \sin(100\pi t) \text{ volts}$$

Solution:

a. Using Thevenin's Theorem, we'll read the impedance from the load side and turn off the voltage sources by replacing them with a shorted wire:



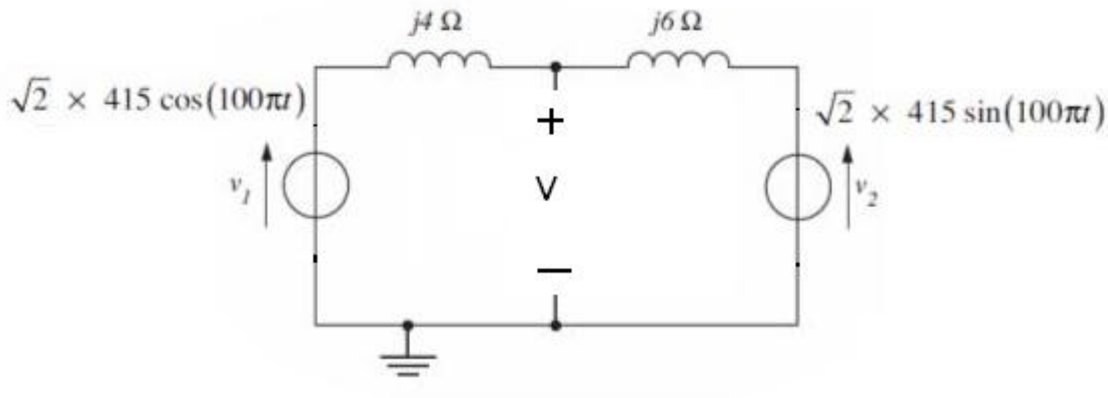
The equivalent impedance is:

$$Z = \frac{(j4)(j6)}{j4 + j6}$$

$$Z = \frac{-24}{j10}\left(\frac{j}{j}\right)$$

$$Z = j2.4\Omega$$

Then we'll return the voltage sources and leave the load terminals open to read the voltage across the load:



Using nodal analysis, we have:

$$I_{L1} = I_{L2}$$

$$\frac{v_1 - v_L}{j4} = \frac{v_L - v_2}{j6}$$

$$\frac{415\sqrt{2}\cos(100\pi t) - v_L}{j4} = \frac{v_L - 415\sqrt{2}\sin(100\pi t)}{j6}$$

$$\frac{415\sqrt{2}\sin(100\pi t + 90) - v_L}{j4} = \frac{v_L - 415\sqrt{2}\sin(100\pi t)}{j6}$$

$$\frac{j415\sqrt{2} - v_L}{j4} = \frac{v_L - 415\sqrt{2}}{j6}$$

$$\left(j415\sqrt{2} - v_L\right)(j6) = \left(v_L - 415\sqrt{2}\right)(j4)$$

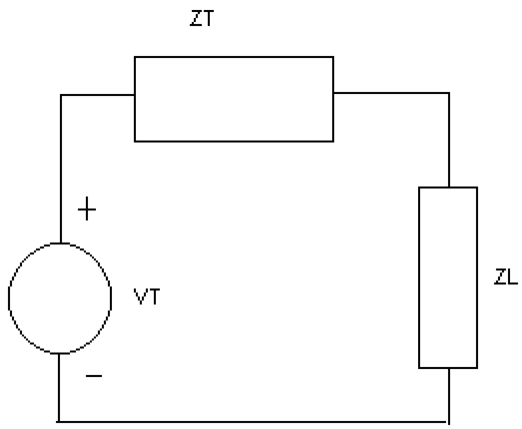
$$-2490\sqrt{2} - j6v_L = j4v_L - j1660\sqrt{2}$$

$$-2490\sqrt{2} + j1660\sqrt{2} = j10v_L$$

$$v_L = 166\sqrt{2} + j249\sqrt{2}$$

$$v_L = 423.22\sin(100\pi t + 56.31°)Volts$$

So the Thevenin model will be:



The equivalent load impedance is equal to:

$R = 50\Omega$

$X = 50\tan\left(\cos^{-1}0.7\right) = 51.01\Omega$

$Z = R + jX = 50 + j51.01$

Computing for the load current, we have:

$Z_{EQ} = Z_T + Z_L$

$Z_{EQ} = Z_T + Z_L$

$Z_{EQ} = j2.4 + 50 + j51.01$

$Z_{EQ} = 50 + j53.41\Omega$

$I_L = \dfrac{V_T}{Z_{EQ}}$

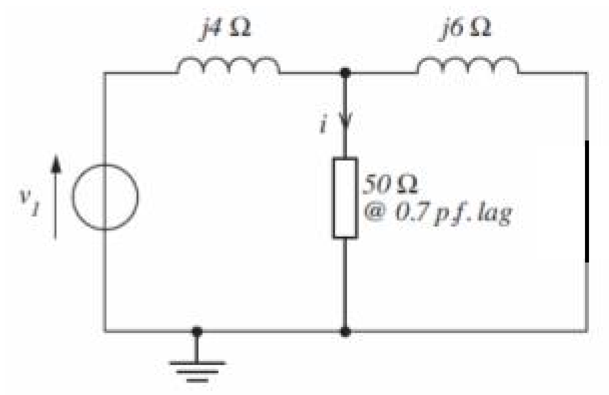
$I_L = \dfrac{423.22\sin\left(100\pi t + 56.31\right)}{50 + j53.41}$

$I_L = \dfrac{423.22\angle 56.31}{73.16\angle 46.89}$

$I_L = 5.78\angle 9.42$

$I_L = 5.78\sin\left(100\pi t + 9.42^\circ\right)A$

b. Using superposition, we first turn off one voltage and compute for the current with the other voltage turned on. So we have:



Solving, we have:

$$v_{L1} = \left(415\sqrt{2}\cos(100\pi t)\right)\left(\frac{\frac{(50+j51.01)(j6)}{50+j51.01+j6}}{j4+\left[\frac{(50+j51.01)(j6)}{50+j51.01+j6}\right]}\right)$$

$$v_{L1} = \left(j415\sqrt{2}\right)\left(\frac{\frac{-306.06+j300}{50+j57.01}}{j4+\left[\frac{-306.06+j300}{50+j57.01}\right]}\right)$$

$$v_{L1} = \left(j415\sqrt{2}\right)\left(\frac{0.31+j5.64}{j4+0.31+j5.64}\right)$$

$$v_{L1} = \left(j415\sqrt{2}\right)\left(\frac{0.31+j5.64}{0.31+j9.64}\right)$$

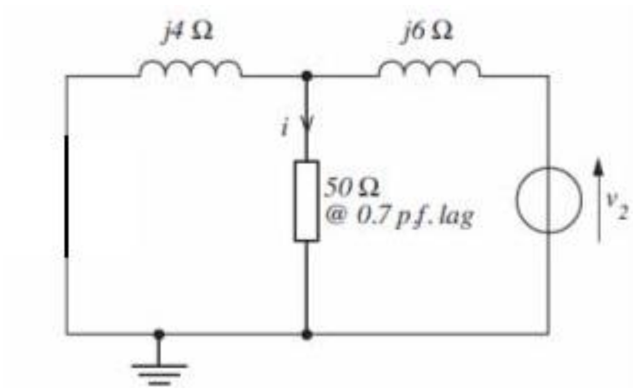
$$v_{L1} = \left(j415\sqrt{2}\right)(0.59-j0.01)$$

$$v_{L1} = 5.87+j346.27$$

$$I_{L1} = \frac{5.87+j346.27}{50+j51.01}$$

$$I_{L1} = 3.52+j3.33A$$

We now turn on the other voltage and turn off the first one:



Solving, we have:

$$v_{L2} = \left(415\sqrt{2}\sin(100\pi t)\right)\left(\frac{\frac{(50+j51.01)(j4)}{50+j51.01+j4}}{j6+\left[\frac{(50+j51.01)(j4)}{50+j51.01+j4}\right]}\right)$$

$$v_{L2} = \left(415\sqrt{2}\right)\left(\frac{\frac{-204.04+j200}{50+j55.01}}{j6+\left[\frac{-204.04+j200}{50+j55.01}\right]}\right)$$

$$v_{L2} = \left(415\sqrt{2}\right)\left(\frac{0.14+j3.84}{j6+0.14+j3.84}\right)$$

$$v_{L2} = \left(415\sqrt{2}\right)\left(\frac{0.14+j3.84}{0.14+j9.84}\right)$$

$$v_{L2} = \left(415\sqrt{2}\right)(0.39-j0.01)$$

$$v_{L2} = 228.89-j5.87$$

$$I_{L2} = \frac{228.89-j5.87}{50+j51.01}$$

$$I_{L2} = 2.18-j2.35A$$

Combining the solved current values, we have the final current value of:

$$I_L = 3.52+j3.33+(2.18-j2.35)$$

$$I_L = 5.7+j0.98$$

$$I_L = 5.78\sin\left(100\pi t+9.76^\circ\right)A$$

Comparing both the Thevenin and the superposition solutions, both are relatively close answers.