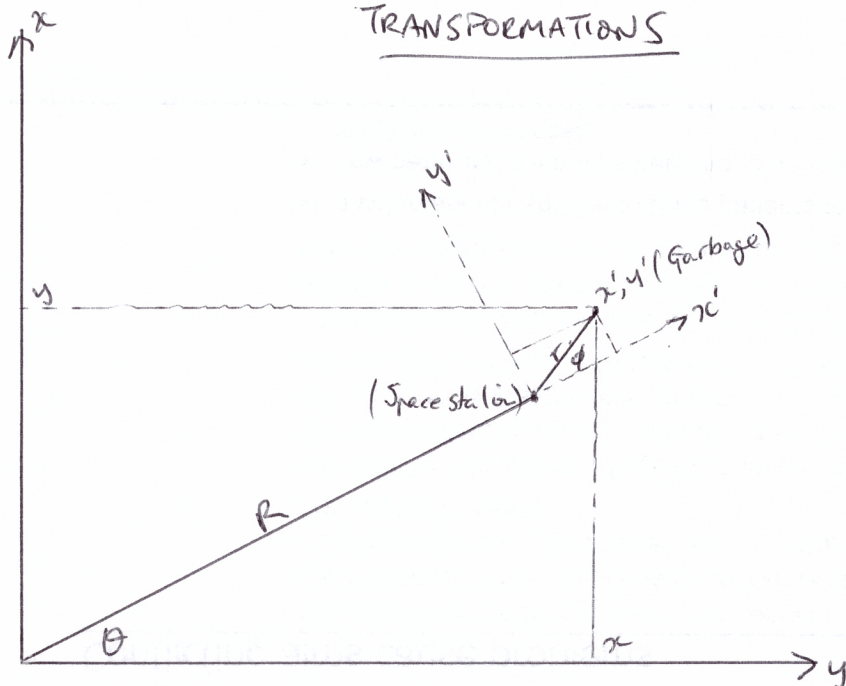


TRANSFORMATIONS



$$x = R \cos \theta + r (\cos \theta + \phi)$$

$$y = R \sin \theta + r (\sin \theta + \phi)$$

$$\rightarrow x' = -R + x \cos \theta + y \sin \theta$$

$$y' = y \cos \theta - x \sin \theta$$

$$x = R \cos \theta + x' \cos \theta - y' \sin \theta$$

$$y = R \sin \theta + x' \sin \theta + y' \cos \theta$$

$$\frac{\partial x}{\partial x'} = \cos \theta$$

$$\frac{\partial x}{\partial y'} = -\sin \theta$$

$$\frac{\partial y}{\partial x'} = \sin \theta$$

$$\frac{\partial y}{\partial y'} = \cos \theta$$

$$\frac{\partial x'}{\partial x} = \cos \theta$$

$$\frac{\partial y'}{\partial y} = \cos \theta$$

$$\frac{\partial y'}{\partial x} = -\sin \theta$$

$$\frac{\partial x'}{\partial y} = \sin \theta$$

fiducial test particle can be neglected except $dx^0/d\tau = 1$. The space components of the equation of geodesic deviation read

$$d^2\xi^k/d\tau^2 + R^k_{0j0}\xi^j = 0. \quad (1.13)$$

Comparing with the conclusions of Newtonian theory, equations (1.5), we arrive at the following information about the curvature of spacetime near a center of mass:

$$\begin{vmatrix} R^{\hat{z}}_{\hat{0}\hat{x}\hat{0}} & R^{\hat{y}}_{\hat{0}\hat{x}\hat{0}} & R^{\hat{z}}_{\hat{0}\hat{z}\hat{0}} \\ R^{\hat{z}}_{\hat{0}\hat{y}\hat{0}} & R^{\hat{y}}_{\hat{0}\hat{y}\hat{0}} & R^{\hat{z}}_{\hat{0}\hat{y}\hat{0}} \\ R^{\hat{z}}_{\hat{0}\hat{z}\hat{0}} & R^{\hat{y}}_{\hat{0}\hat{z}\hat{0}} & R^{\hat{z}}_{\hat{0}\hat{z}\hat{0}} \end{vmatrix} = \begin{vmatrix} m/r^3 & 0 & 0 \\ 0 & m/r^3 & 0 \\ 0 & 0 & -2m/r^3 \end{vmatrix} \quad (1.14)$$

(units cm^{-2}). Here and henceforth the caret or “hat” is used to indicate the components of a vector or tensor in a local Lorentz frame of reference (“physical components,” as distinguished from components in a general coordinate system). Einstein’s theory will determine the values of the other components of curvature (e.g., $R^{\hat{z}}_{\hat{z}\hat{z}\hat{z}} = -m/r^3$); but these nine terms are the ones of principal relevance for many applications of gravitation theory. They are analogous to the components of the electric field in the Lorentz equation of motion. Many of the terms not evaluated are analogous to magnetic field components—ordinarily weak unless the source is in rapid motion.

This ends the survey of the effect of geometry on matter (“effect of curvature of apple in causing geodesics to cross”—especially great near the dimple at the top, just as the curvature of spacetime is especially large near a center of gravitational attraction). Now for the effect of matter on geometry (“effect of stem of apple in causing dimple”)!

§1.7. EFFECT OF MATTER ON GEOMETRY

The weight of any heavy body of known weight at a particular distance from the center of the world varies according to the variation of its distance therefrom; so that as often as it is removed from the center, it becomes heavier, and when brought near to it, is lighter. On this account, the relation of gravity to gravity is as the relation of distance to distance from the center.

AL KHĀZINĪ (Merv, A.D. 1115), *Book of the Balance of Wisdom*

Figure 1.12 shows a sphere of the same density, $\rho = 5.52 \text{ g/cm}^3$, as the average density of the Earth. A hole is bored through this sphere. Two test particles, *A* and *B*, execute simple harmonic motion in this hole, with an 84-minute period. Therefore their geodesic separation ξ , however it may be oriented, undergoes a simple periodic motion with the same 84-minute period:

$$d^2\xi^j/d\tau^2 = -\left(\frac{4\pi}{3}\rho\right)\xi^j, \quad j = x \text{ or } y \text{ or } z. \quad (1.15)$$