$$3V(D) = 3\int_0^{2\pi}\int_0^1\int_{r^2}^{\sqrt{2-r^2}} dz \cdot r\,dr\,d\theta$$

$$= 3\int_0^{2\pi}\int_0^1 \left[z\right]_{r^2}^{\sqrt{2-r^2}} r\,dr\,d\theta$$

$$= 3\int_0^{2\pi}\int_0^1 \left(\sqrt{2-r^2} - r^2\right)\cdot r\,dr\,d\theta$$

$$= 3\int_0^{2\pi}\int_0^1 \left(r\sqrt{2-r^2} - r^3\right) dr\,d\theta$$

$$= 3\int_0^{2\pi} d\theta \int_0^1 \left(r\sqrt{2-r^2} - r^3\right) dr$$

$$= 6\pi\int_0^1 r\sqrt{2-r^2}\,dr - 6\pi\int_0^1 r^3\,dr,$$ Let $w = 2 - r^2$. Then, $\frac{dw}{dr} = -2r \Rightarrow dr = \frac{dw}{-2r}$

$$= \frac{6\pi}{-2}\int_2^1 \sqrt{w}\,dw - \frac{6\pi}{4}\left[r^4\right]_0^1$$

$$= +3\pi\int_1^2 w^{1/2}\,dw - \frac{3\pi}{2}\left(1^4 - 0^4\right)$$

$$= 3\pi \cdot \frac{2}{3}\left[w^{3/2}\right]_1^2 - \frac{3\pi}{2}$$

$$= 2\pi\left(2^{3/2} - 1\right) - \frac{3\pi}{2}$$

$$= \pi\left[2\left(2^{3/2} - 1\right) - \frac{3}{2}\right]$$