$$a = \frac{\Delta}{4p} \quad , \quad x = \frac{2p\alpha N_e}{\Delta} \quad , \quad \beta_{\pm} = \pm(x - \cos 2\theta) \tag{1}$$

$$\begin{aligned}
&det(H_M - Im) = 0 \\
&a^4\Big[(x - \cos 2\theta - m)(-x + cos2\theta - m) - sin^2 2\theta\Big] \\
=&a^4\Big[(\beta_+ - m)(\beta_- - m) - sin^2 2\theta\Big] = a^4\Big[\beta_+\beta_- + m^2 - m(\beta_- + \beta_+) - sin^2 2\theta\Big] = 0 \\
\Rightarrow& m^2 - (\beta_+ + \beta_-)m + \beta_+\beta_- - sin^2 2\theta = 0
\end{aligned} \tag{2}$$

$$\beta_+ + \beta_- = x - \cos 2\theta - x + \cos 2\theta = 0 \tag{3}$$

$$\beta_+\beta_- = (x - \cos 2\theta)(-x + \cos 2\theta) = -x^2 - \cos^2 2\theta + 2x\cos 2\theta \tag{4}$$

And so:

$$m^2 = sin^2 2\theta + (x - \cos 2\theta)^2 \Rightarrow m = \pm\sqrt{(x - \cos 2\theta)^2 + \sin^2 2\theta} \tag{5}$$