Performing a Taylor expansion around $t = 0$, we have shown that

$$\begin{aligned} g(\beta,\beta)(t\alpha) = g_0(\beta,\beta) + \frac{t^2}{3}g_0(R_\alpha(\beta),\beta) + \frac{t^3}{6}g_0(D_tR_\alpha(\beta),\beta) \\ + \frac{t^4}{20}g_0(D_t^2R_\alpha(\beta),\beta) + \frac{2t^4}{45}g_0(R_\alpha(\beta),R_\alpha(\beta)). \end{aligned} \tag{4.22}$$

We let $\alpha = (x^i/t)\partial_i$, and $\beta = \beta^j\partial_j$. The first term on the right hand side of (4.22) is simply

$$g_0(\beta,\beta) = \delta_{ij}\beta^i\beta^j. \tag{4.23}$$

The second term on the right hand side of (4.22) is

$$\begin{aligned} \frac{t^2}{3}g_0(R_\alpha(\beta),\beta) &= \frac{t^2}{3}g_0(R(\alpha,\beta)\alpha,\beta) && (4.24)\\ &= \frac{1}{3}g_0(R(x^k\partial_k,\beta^i\partial_i)x^l\partial_l,\beta^j\partial_j) && (4.25)\\ &= \frac{1}{3}x^kx^lR_{kil}{}^m\delta_{mj}\beta^i\beta^j && (4.26)\\ &= \frac{1}{3}R_{kijl}x^kx^l\beta^i\beta^j && (4.27) \end{aligned}$$

Also, since the Christoffel symbols vanish at p, covariant derivatives are just ordinary partial derivatives. We also have that $\partial_t = (x^i/t)\partial_i$. The third term on the right hand side of (4.22) is

$$\begin{aligned} \frac{t^3}{6}g_0(D_tR_\alpha(\beta),\beta) &= \frac{t^3}{6}g_0(\partial_tR(\alpha,\beta)\alpha,\beta) && (4.28)\\ &= \frac{1}{6}g_0(x^m\partial_mR(x^k\partial_k,\beta^i\partial_i)x^l\partial_l,\beta^j\partial_j) && (4.29)\\ &= \frac{1}{6}\nabla_mR_{kil}{}^p\delta_{pj}x^mk^kx^l\beta^i\beta^j && (4.30)\\ &= \frac{1}{6}\nabla_mR_{kijl}x^mk^kx^l\beta^i\beta^j && (4.31) \end{aligned}$$