NOTATIONAL “VIOLATION” IN CALCULUS

“Derivatives are fractions”. Any number can in fact be represented in fraction form.
By definition, a derivative is the gradient of the tangent to the graph and gives the graph’s instantaneous direction at any point. The tangent gradient is the “slope of a line” which has a vertical component divided by a horizontal component.
It is this that makes a derivative a fraction. The numerator is not “dx “ and the denominator is not “dy” in the accepted sense of fraction., since what matters is the “ratio” of “dy” to “dx”, not the “values” of “dy” and “dx”.

$\dfrac{dy}{dx}$ is the notation that signifies $\displaystyle\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ which is the tangent gradient.

It refers to an “infinitesimal change in y” divided by an “infinitesimal change in x”.

These infinitesimal values arise as we have picked 2 points on the graph of the function, and written the line slope or gradient using them BUT we “tilt” the line until it is a tangent to the graph. Hence the right-angled triangle we were using to write the line gradient (between 2 points on the curve) has DISAPPEARED!! or has it??
Consider this Δ to have become reduced to the molecular level! After all, the tangent contains the hypotenuse of this Δ so it’s really still there though we can no longer see it due to the fact that we tilted the tangent until the second point disappeared, but we still have a “Gulliver’s Travels” situation. Now imagine magnifying this Δ until it’s clearly visible again.

Therefore $\dfrac{dy}{dx}$ is the tangent of the angle the tangent makes with the horizontal axis.

It’s a fraction at any value of the horizontal variable, a varying fraction in the case of curves… true, but a fraction.

Once the limit as $\Delta x \to 0$ has been evaluated, $\dfrac{dy}{dx} = \dfrac{\Delta y}{\Delta x}$ for the tangent.

$\dfrac{\Delta y}{\Delta x}$ pertains to “increments” and we distinguish between the tangent and the curve.

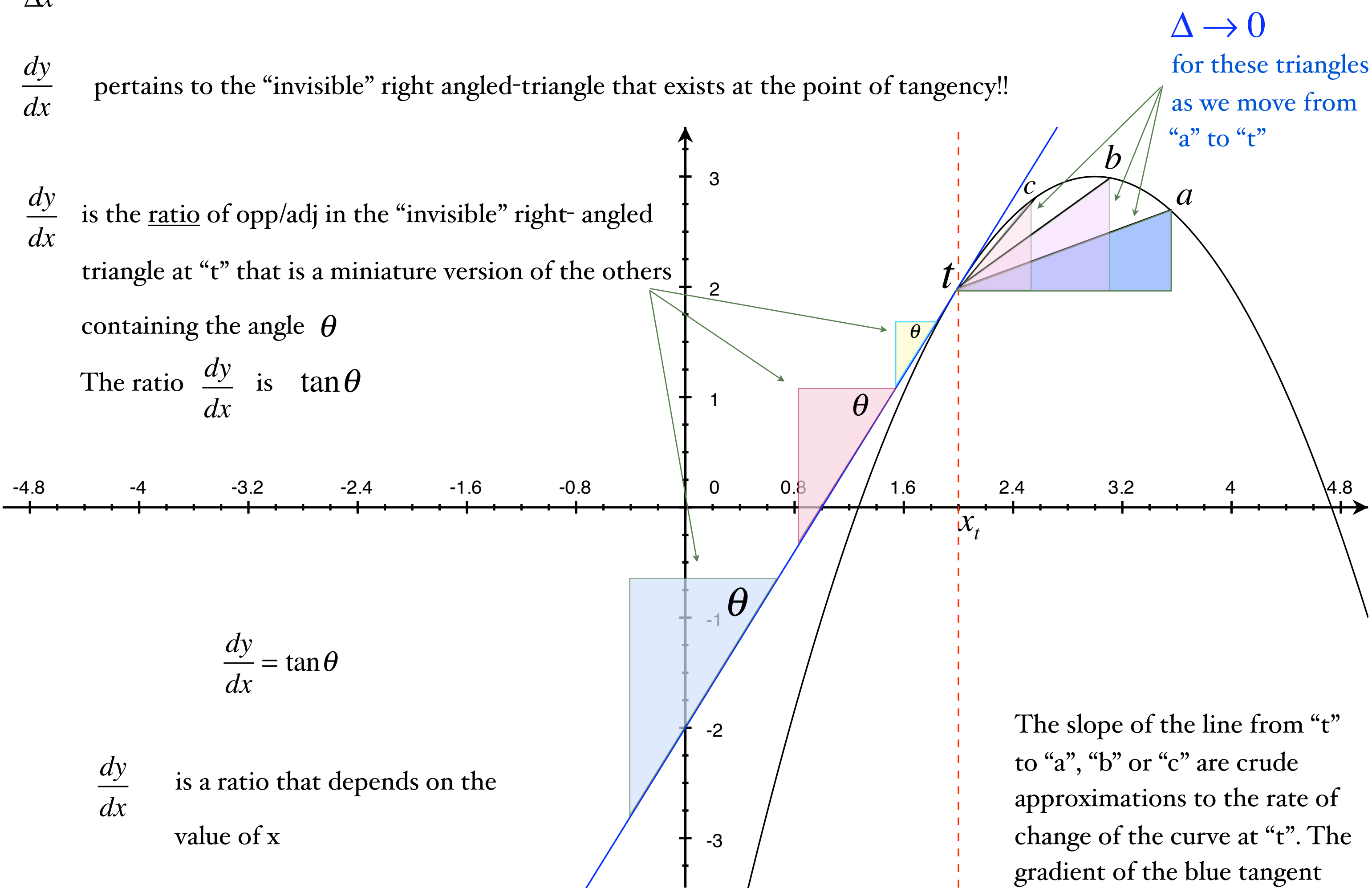
$\dfrac{dy}{dx}$ pertains to the “invisible” right angled-triangle that exists at the point of tangency!!

$\dfrac{dy}{dx}$ is the ratio of opp/adj in the “invisible” right- angled triangle at “t” that is a miniature version of the others containing the angle θ

The ratio $\dfrac{dy}{dx}$ is $\tan\theta$

$$\frac{dy}{dx} = \tan\theta$$

$\dfrac{dy}{dx}$ is a ratio that depends on the value of x



The slope of the line from “t” to “a”, “b” or “c” are crude approximations to the rate of change of the curve at “t”. The gradient of the blue tangent gives us it’s rate of change at $f(x_t)$

Derivatives are "derived" from the function equation by joining two points on the function and writing the gradient of the line between these two points because that gives us the rate of change of the function "over that interval Δx . Only the gradient of the tangent gives us the "instantaneous rate of change".

This is called $\frac{dy}{dx}$ for the following reason and this is essential to understand!!

Many students seem to think that $\frac{dy}{dx}$ is merely calculus notation but it's not!! It was brilliantly thought through and we can appreciate it by peering through through the microscope of our mind's eye, to see the beauty of it.

$\frac{f(x+\Delta x)-f(x)}{\Delta x}$ is the gradient of the line from "t" to "a", "b" or "c" etc, where Δx is decreasing and so $f(x+\Delta x)$ approaches $f(x)$ and therefore "in these terms" the gradient of the tangent at "t" is $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ giving rise to the "illusion" that we have a "divide by zero" situation!

The "Bermuda Triangle" situation we appear to have here is an illusion. It appears that we have a "missing triangle" due to what is happening as we move from "a" to "t" backwards along the curve. It becomes delusional to the extent that we focus in on the algebra without staying focused on what is taking place geometrically. The vanishing triangle has side ratios identical to the side ratios of the triangles on the left of the tangent with angle θ. The tangent of the angle θ is the gradient of the tangent, which graphically represents the instantaneous rate of change of the function at x_t specifically.

Hence, the <u>ratio</u> $\frac{dy}{dx}$ equals $\tan\theta$ when $x = x_t$

The value of this ratio is defined when you choose a specific value of x.

Therefore you may treat $\frac{dy}{dx}$ *and* $\frac{dx}{dy}$ as inverse ratios IF the gradients are calculated at the same value of x or y for $y = f(x)$ *or* $x = f(y)$ or if the function is periodic and the points are a cycle apart.

CLEARLY we cannot say $\frac{dy}{dx} = \frac{dy}{dx}$ at parts of the curve that have different gradients!! In this case we simply have not been specific enough! $\frac{dy}{dx}$ refers to the gradient in general unless we are being specific about position.

When we are at the same position $\frac{dy}{dx} = \frac{1}{\left(\frac{dy}{dx}\right)}$ and there is no ambiguity about the derivative ratios.

Also, when we introduce $\frac{dx}{dx}$ as in the "chain rule", to differentiate the function of a function, we may swop the denominators since we are operating at a specific position for the new components of the derivative and the mathematics of that technique is reflected in the adjustments within first principles.

Imagine you are driving in your car. If you don't have a speed dial, you may calculate your speed using a time interval or a distance interval but your speed will be distance interval over time interval. You will always be calculating an average speed of course. If the time interval is zero, you will be able to calculate your instantaneous speed with a computer and sensors.

Wait a minute!!! Did you say a "TIME INTERVAL OF ZERO?" Isn't time in the denominator position, which gives us a divide by zero? That's the end of that! How can we perform measurements in zero time?

When it comes to algebraic methods, we resolve the divide by zero, by ensuring we have the term that approaches zero as a factor of the denominator or using some other valid means, thereby cancelling it.

Now, as we drive along in our car, our instantaneous speed is represented exactly by the tangent to the graph of distance versus time IF we could take enough measurements to draw a continuous graph, so although the triangle we were using "imploded", it resurrected itself on the tangent. We were never dividing by zero, it only looked that way. We simply ended up with a problem of "scale" as we were deriving the derivative from the function itself.

$\frac{dy}{dt}\frac{dt}{dy} = \frac{dy}{dt} \Big/ \frac{dy}{dt} = 1$ for a specific point in time, not for different times driving at different speeds!

nor can you do this for "partial" derivatives.