$$\vec{r} - \vec{r}' = z\hat{z} - r'\hat{r}$$

$$\hat{r} = \sin\theta'\cos\varphi'\hat{x} + \sin\theta'\sin\varphi'\hat{y} + \cos\theta'\hat{z}$$

$$\vec{r} - \vec{r}' = r'\sin\theta'\cos\varphi'\hat{x} + r'\sin\theta'\sin\varphi'\hat{y} + r - r'\cos\theta\,\hat{z}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_0 e^{-kr}}{(z^2 + r'^2 - 2rz'\cos\theta)^{3/2}}\Big(r'\sin\theta'\cos\varphi'\hat{x} + \quad (\to 0)$$

$$+ r'\sin\theta'\sin\varphi'\hat{y} + r - r'\cos\theta'\hat{z}\Big)r'^2\sin\theta'\,dr'd\varphi'd\theta'$$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\int \rho_0 e^{-kr}\,\frac{(r - r')r'^2\sin\theta'\cos\theta'\,d\theta'}{(z^2 + r'^2 - 2rz'\cos\theta)}$$