Math 225 - Differential Equations I
Assignment # 1
due January 16th, 9:30am

Instructions: You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Sloppy or incorrect use of technical terms will lower your mark.

Handing in the Assignment: On the due date, your name should be on three separate documents:

Document 1: Problems from Part 1(a) (one per group of 1-5)

Document 2: Problems from Part 1(b) (one per group of 1-5)

Document 3: Research Article (one per group of 1-3)

Part 1(a)

Problems in this section may be done with up to 4 other classmates. If you collaborate with classmates, the group should hand in one document with all contributing names at the top.

1. Fill in the table below (if the blank is in the 1st column, invent an example of your own or find one in the text). Note that "ind. var." means "independent variable" and "dep. var." means "dependent variable". The first row has been filled out for you.

equation	linear	order	ind. var.	dep. var.
$\frac{d^2x}{dt^2} + \sin(t)x = 0$	yes	2	t	x
$(1-x)\frac{d^3y}{dx^3} - 4x\frac{dy}{dx} + 5y = 0$		3		
$t\frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^4 + x = 0$				x
	no	1	r	P
$\sin(\theta)\frac{d^4y}{d\theta^4} - \cos(\theta)\frac{dy}{d\theta} = 2$			θ	
$\frac{dR}{dt} = -\frac{k}{R^2}$				

2. Verify that the family of functions

$$P(t) = \frac{Ce^t}{1+Ce^t},$$

where C is an arbitrary constant, is a solution of the differential equation

$$\frac{dP}{dt} = P(1-P)$$

(this is the logistic growth equation we saw in class). Then determine the value of C needed for this solution to satisfy the initial condition $P(0) = 10$.

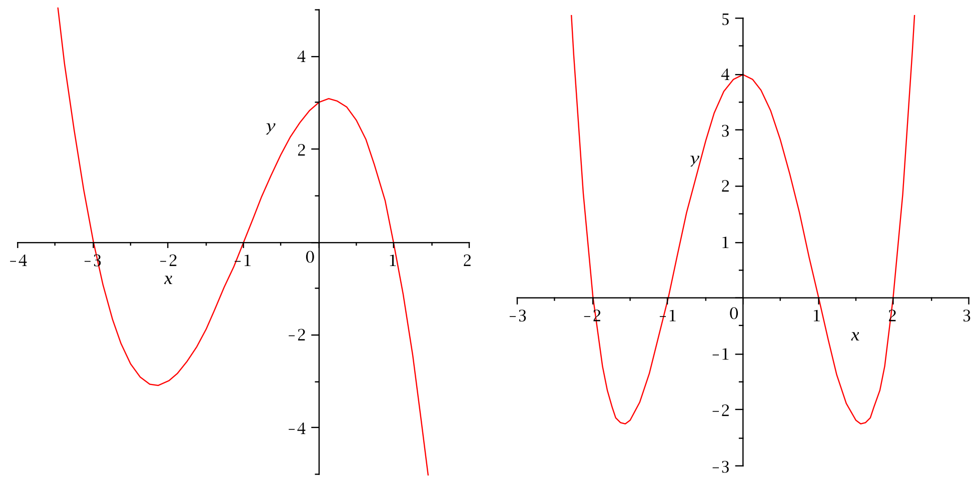
3. Consider the initial value problem

$$y' = \frac{g(t)y}{1+p(t)y}, \quad y(0) = 3.$$

Suppose that $p(t)$ and/or $g(t)$ have discontinuities at $t = -5$, $t = 1$, and $t = 4$ but are continuous for all other values of t. What is the largest interval (a, b) on which Theorem 1 (section 1.2) guarantees the existence of a unique solution to the initial value problem?

4. Make up a differential equation that you feel confident possesses only the trivial solution $y = 0$. Explain your reasoning.

5. Figure 1 shows (a) the graph of $f(N)$ and (b) the graph of $f(t)$. By hand, sketch two direction fields, one for $dN/dt = f(N)$ and one for $dN/dt = f(t)$, on the grid $-3 \le t \le 3$.



(a) graph of f(N) (b) graph of f(t)

Figure 1: Direction fields for Problem 5.

6. Often a radical change in the form of the solution of a differential equation corresponds to a very small change in either the initial condition or the equation itself. Using separation of variables, find a solution to each of the three initial value problems below, and use a graphing utility (such as Maple) to plot the graph of each solution. Compare each solution curve in a neighbourhood of (0,1) and comment.

$$\begin{aligned}
\frac{dy}{dx} &= (y-1)^2, & y(0) &= 1 \\
\frac{dy}{dx} &= (y-1)^2, & y(0) &= 1.01 \\
\frac{dy}{dx} &= (y-1)^2 + 0.01, & y(0) &= 1
\end{aligned}$$

Part 1(b)

Problems in this section may be done with up to 4 other people. If you collaborate with classmates, the entire group should hand in one completed document with all contributing names on top.

7. Group Project D (The Phase Line) from Chapter 1, parts (f)-(h).

Part 2

This section of the assignment may be done with up to 2 other classmates.

We will be spending the next two weeks studying first order differential equations. The equation for exponential growth or decay is the simplest of these, and has a multitude of applications. Go to the web, the library and your textbooks to gather information about exponential growth, and write a one to two page article on the topic. Your article should be formatted as follows:

paragraph 1 Introduce the differential equation for exponential growth (explain what it is and how it works), and derive its solution. Close with a sentence that leads into the second paragraph.

paragraph 2 Present your first example of an application of the differential equation for exponential growth. Explain the context, what the equation means in that context, and how the solution is useful (or not!).

paragraph 3 Present a second example of an application of the differential equation for exponential growth. This second example must be drawn from a different field than the first example. Again, explain the context of your second example, what the differential equation means in that context, and how the solution is useful (or not!).

paragraph 4 Explain what exponential growth implies, and why an understanding of exponential growth is important. Think about the following questions when writing this paragraph:

1. If the human population is growing exponentially, what are the implications for the planet?
2. Investment saving strategies are based on exponential growth of the principal deposited. Do you think the growth of wealth can continue indefinitely? What are the implications of either a yes or no answer?
3. Invasive species (such as eurasian milfoil, purple loosestrife, gypsy moth, and zebra mussels) are particularly damaging because they demonstrate exponential growth after initiation of the invasion. Why do you think it is possible for these invaders to grow exponentially?

Hint: Compare exponential growth to geometric growth. Also, see the YouTube video http://www.youtube.c

You must provide references in the scientific style for all statements you make. Your reference list must include at least six different references of which at most one may be a website. A sample article is provided to help you. Note that this sample article is about a different topic, the derivative, with which you should be familiar. So it is not formatted exactly the same way your article on the differential equation for exponential growth should be formatted. You can see however, how the referencing is done, and how one should write about a mathematical topic.

Your article should be at most two pages long (not including references), double-spaced, in 12-point font. Hand in your article as a document separate from your work for Part 1 of this assignment. This part of the assignment may be done with another person. If you collaborate with a classmate, the two of you should hand in one assignment with both of your names on it.