On partitioning an even number into a pair of (relative) primes

Let's look at this with the help of an **example**:

$$m = 68 \equiv \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} (\mathrm{mod} \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix})$$

Splitting the parenthesis $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ into two additive components $\begin{pmatrix} i \\ k \\ l \end{pmatrix}$ *(* $0 < i < 3,\ 0 < k < 5,\ 0 < l < 7$ *)*

yields a structure which consists of $r = \ (3-2)*(5-2)*(7-2) = 15$ components forming the eight different partitions (seven asymmetric plus one symmetric) as follows:

$$\begin{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 1 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} \end{bmatrix} \equiv \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} (\mathrm{mod} \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}) \qquad (1)$$

To each component, a unique number (which may be called a *relative prime*)

$$v\ (0 < v < 2 \times 3 \times 5 \times 7), \qquad (2)$$

can be assigned, juxtaposing to (1) the *isomorphic structure*:

$$\begin{bmatrix} 1 + 67 \equiv 127 + 151 \equiv 169 + 109 \\ 121 + 157 \equiv 37 + 31 \equiv 79 + 199 \\ 181 + 97 \equiv 139 + 139 \end{bmatrix} \equiv 68(\mathrm{mod}(2 \times 3 \times 5 \times 7)), \qquad (3)$$

all numbers shown in the bracket on the left containing none of the divisors $3,5,7$. For $m=68\ (7^2<68<11^2)$, it includes all the partitions into prime numbers (including $68=1+67$ and excluding $68=7+61$, for evident reasons).

The structure (1) can be extended by including one higher prime number after another.

Thus, for $m=122\ (11^2<122<13^2)$ one finds $r=(3-2)\times(5-2)\times(7-2)\times(11-2)=135$ relative primes $v\ \ (0<v<2*3*5*7*11)$, forming 68 pairs (67 asymmetric and one symmetric), among them the four partitions into prime numbers: 13+109, 19+103, 43+79, 61+61.

And, for $m=176\ (13^2<176<17^2)$, one finds
$r=(3-2)\times(5-2)\times(7-2)\times(11-1)\times(13-2)=1650$ relative primes: v
$(0<v<2\times3\times5\times7\times11\times13)$,
forming 825 asymmetric pairs, among them four partitions into prime numbers: 19+157, 37+139, 67+109, 73+103, 79+97.

Conclusion

Any even number $m\ \ (m\geq 10),\ (p_n^2<m<p_{n+1}^2)$ *(A)*

can be partitioned into

$$r_n(m)=\prod_{\substack{i=1\\p_1=3}}^{i=n}(p_i-k_i)$$ *(* $k_i=1$ *for* $m\equiv 0\ (\mathrm{mod}\ p_i)$, *otherwise* $k_i=2$ *)* *(B)*

relative primes $v\ (0<v<2\times\prod_{\substack{i=1\\p_1=3}}^{i=n}p_i)$ *(C)*

and will admit

- $\frac{1}{2}r_n(m)$ *asymmetric partitions (* $r_n(m)$ *being an even number)*
- $\frac{1}{2}(r_n(m)-1)$ *asymmetric partitions, plus one symmetric (if* $r_n(m)$ *is an odd number)*

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With the exception of partitions which contain one of the prime numbers $p_i\ \ (p_1=3\leq p_i\leq p_n)$, *all partitions of* m *into prime numbers will be found among these partitions (one at least).*