PROBLEMS

10-1 Simple Harmonic Motion

1. A mass on the end of a spring is stretched a distance x_0 from equilibrium and released. At what distance from equilibrium will it have (a) velocity equal to half its maximum velocity and (b) acceleration equal to half its maximum acceleration?

2. The position of a simple harmonic oscillator as a function of time is given by $x = 3.8cos(\frac{7\pi}{4} + \frac{\pi}{6})$ where t is in seconds and x in centimeters. Find (a) the period and frequency, (b) the position and velocity at t = 0, and (c) the velocity and acceleration at t = 2.5s.

3. The amplitude of the motion of a mass attached to a spring is x_m = 2.84 m, while the maximum speed of the mass is 4.36 m/s. What is the period of the motion?

4. A harmonic oscillator operates at a frequency of 58.966 Hz. What is the amplitude for which the maximum acceleration is 103.57 m/s^2?

10-3 The Force Law for Simple Harmonic Motion

5. A uniform meter stick of mass M is pivoted on a hinge at one end and held horizontal by a spring with spring constant k attached at the other end (Fig. 10-20). If the stick oscillates up and down slightly, what is the frequency? [Hint: Write a torque equation about the hinge.]

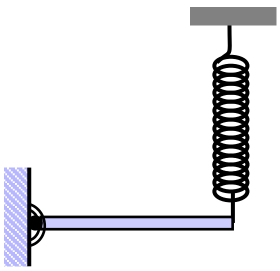


Fig. 10-20 Problem 5.

6. A mass m is connected to two springs, with spring constants k_1 and k_2, in two different ways as shown in Fig. 10-21a and 10-21b. Show that the period for the configuration in part (a) is given by $T = 2\pi\sqrt{m(\frac{1}{k_1} + \frac{1}{k_2})}$ and for that in part (b) is given by $T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$.

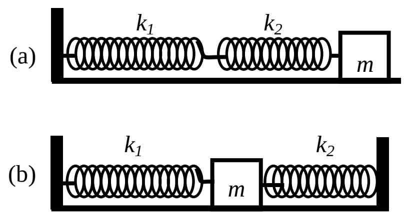


Fig. 10-21 Problem 6.

7. At a certain harbor, the tides cause the ocean surface to rise and fall a distance d (from highest level to lowest level) in simple harmonic motion, with a period of 12.5h. How long does it take for the water to fall a distance $d/4$ from its highest level?

8. A block is on a horizontal surface (a shake table) that is moving back and forth horizontally with simple harmonic motion of frequency 2.0 Hz. The coefficient of static friction between block and surface is 0.50. How great can the amplitude of the SHM be if the block is not to slip along the surface?

9. Two particles execute simple harmonic motion along a common straight-line segment of length A. Each particle has a period of 1.5 s, but they differ in phase by $\pi/6$ rad. (a) How far apart are they (in terms of A) 0.50 s after the lagging particle leaves one end of the path? (b) Are they then moving in the same direction, toward each other, or away from each other?

10-4 Energy in Simple Harmonic Motion

10. A mass m attached to the end of a spring is released from rest at $t = 0$ from an extended position x_{m}. The mass $m = 0.2$ kg, and $k = 1$ N/m. After 0.5 s, the speed of the mass is measured to be 1.5 m/s. Calculate x_m, the maximum speed of the motion, and the total energy.

11. When the displacement in SHM is one-half the amplitude x_m, what fraction of the total energy is (a) kinetic and (b) potential energy? (c) At what displacement, in terms of the amplitude, is the energy of the system half kinetic energy and half potential energy?

12. If one vibration has 10 times the energy of a second one of equal frequency, but the first's spring constant k is twice as large as the second's, how do their amplitudes compare?

13. A mass of 1.2 kg, attached to a spring, is in SHM along the x-axis, and its period is $T = 2.5$ s. If the total energy of the spring and mass is 2.7 J, what is the amplitude of the oscillation?

14. A point mass m on a turntable that rotates with angular frequency ω is located a distance d from the center of the turntable. Show that the energy of the point mass is the sum of the energies of the harmonic motions in the x- and y-directions.

10-5 An Angular Simple Harmonic Oscillator

15. A meter stick is hung at its center from a thin wire (Fig. 10-22). It is twisted and oscillates with a period of 6.0 s. The meter stick is sawed off to a length of 70.0 cm. This piece is again balanced at its center and set in oscillation. With what period does it oscillate?

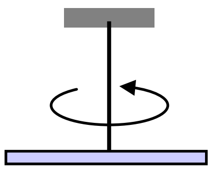


Fig. 10-22 Problem 15.

16. The balance wheel of a watch is a thin ring of radius 0.95 cm and oscillates with a frequency of 3.0 Hz. If a torque of 1.1×10^{-5} m·N causes the wheel to rotate 60°, calculate the mass of the balance wheel.

10-6 Pendulums

17. A simple pendulum 1.20 m long is suspended in a place where g is 9.82 m/s^2. What is the period of the pendulum?

18. The difference in temperature between summer and winter causes the length of the pendulum in a clock to change by one part in 20000. What time-difference error will this make in one day?

19. A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through the stick a distance d from the 50 cm mark. The period of the oscillation is 2.5 s. Find d.

20. In Fig. 10-23, a physical pendulum consists of a uniform solid disk (with mass M and radius R) supported in a vertical plane by a pivot located a distance d from the center of the disk. The disk is displaced by a small angle and released. Find an expression for the period of the resulting simple harmonic motion.

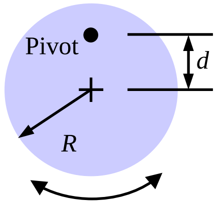


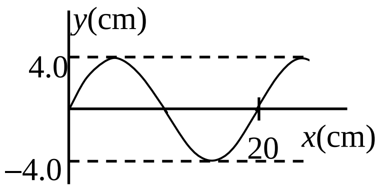
Fig. 10-23 Problem 20.

21. A pendulum is formed by a long thin rod of length L and mass m about a point in the rod that is a distance d above the center of the rod. (a) Find the period of this pendulum in terms of d, L, m, and g, assuming small-amplitude swinging. What happens to the period if (b) d is decreased, (c) L is decreased, or (d) m is decreased?

22. What is the frequency of a simple pendulum 2.0 m long (a) in a room, (b) in an elevator accelerating upward at a rate of 2.0 m/s^2, and (c) in free fall?

Problems

1. $0.866x_0$; (b) $0.500x_0$.



2. (a) (8/7) s, 0.875 Hz; (b) 3.3 m, –10.4 cm/s; (c) –20.7 cm/s, 15.0 cm/s.

3. 4.09 s.

4. 7.5×10^{-4} m.

5. $f = \frac{1}{2\pi}\sqrt{3k/M}$.

6. Proof problem.

7. 2.08h.

8. 3.1 cm.

9. (a) $0.183A$; (b) same direction.

10. 0.74 m, 1.67 m/s. 0.27 J.

11. (a) 3/4; (b) 1/4; (c) $\pm x_m/\sqrt{2}$.

12. $x_{m1} = 2.24x_{m2}$.

13. 0.84m.

14. Proof problem.

15. 3.5 s.

16. 0.33 g.

17. 2.20 s.

18. 2.20 s.

19. 5.6 cm.

20. $T = 2\pi\sqrt{(R^2 + 2d^2)/2gd}$.

21. (a) $2\pi\sqrt{(L^2 + 12d^2)/12gd}$;

(b) Increases for $0 < d < L/\sqrt{12}$;

Decreases for $L/\sqrt{12} < d < L/2$;

(c) Increases; (d) no change.

22. (a) 0.35 Hz; (b) 0.39 Hz; (c) 0.