1 Proof that BEDF is a parallelogram

Let O be the intersection of AC and BD. Because ABCD is a parallelogram, the diagonals AC and BD bisect each other, implying OB = OD. On the other hand, because AECF is also a parallelogram, its diagonal AC and EF also bisect each other, implying OE = OF. From the two conclusion above, we see that OB = OD and OE = OF, meaning that BD and EF bisect each other. Therefore, BEDF is a parallelogram. This leads to the property that $BF \parallel ED$, which will be used in the main proof.

2 Proof the circumcircles intersect at a single point

From the hypothesis, draw the circumcircles of the triangles AEB and CED. The two circles will intersect at E and another point called K. Let J be the intersection of CF and BE.

We have AKEB and CEDK are cyclic quadilaterals. We shall prove that BFKC and AFDK are also cyclic quadilateral, implying K is on the circumcircles of triangles BFC and DFA as well.

Since AKEB is a cyclic quadilateral, we have $\angle EKB \cong \angle EAB$ as the two angles look at the same arc EB. (1)

On the other hand, $\angle EAB \cong \angle FCD$, because $AB \parallel CD$ and $AE \parallel CF$. (2)

From (1) and (2), it can be concluded that $\angle EKB \cong \angle FCD$. (3)

Next, because CEDK is also a cyclic quadilateral, we have $\angle EKC \cong \angle EDC$. (4)

Add (3) and (4) together, we have: $\angle BKC = \angle EKB + \angle EKC = \angle FCD + \angle EDC$. But on the other hand, $\angle FJD$ is the external angle of triangle DJC, meaning that $\angle FJD = \angle FCD + \angle EDC$. Therefore we have $\angle BKC \cong \angle FJD$.

Furthermore, we have $\angle BFC \cong \angle FJD$, because the two angle are alternate interior angles of the two parallel lines BF and ED. Therefore, this leads to $\angle BKC \cong \angle BFC \cong \angle FJD$. Because $\angle BKC \cong \angle BFC$ and they both look at the arc BC, we see that quadilateral BFKC is a cyclic quadilateral, meaning that K is on the circumcircle of triangle BFC.

Using a similar reasoning, we can also prove that AFDK is also a cyclic quadilateral, implying that K is on the circumcircle of triangle ADF.