

TOTAL = 92

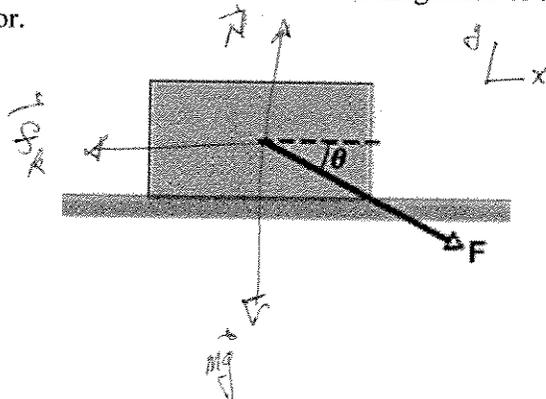
Name

KEY

Date

**BE SURE TO READ THIS FIRST:** Work these problems on separate sheets of paper and staple this sheet to the front. Write only on one side of each page, and box in your final answers. You must show your work or give explanations for *all* answers. I give no credit for unsupported answers. Give all answers to *no more than three* significant figures, and include appropriate units. I do give partial credit, but *only* if I can follow your work, so be as clear as possible about what you are doing.

1. [12 pts] A 3.40 kg block, initially in motion, is pushed along a horizontal floor by a force  $F$  of magnitude 20 N at an angle  $\theta = 30^\circ$  with the horizontal. The coefficient of kinetic friction between the block and floor is 0.250. Calculate the magnitude of the frictional force on the block from the floor.



(a)  $F \cos \theta - \frac{f_k}{k} = ma$   
not needed here.

(b)  $N - mg - F \sin \theta = 0$   
 $N = mg + F \sin \theta$

$\frac{f_k}{k} = \mu_k N$   
 $= \mu_k (mg + F \sin \theta)$   
 $= 0.250 (3.40 \text{ kg} \times 9.8 \text{ m/s}^2$

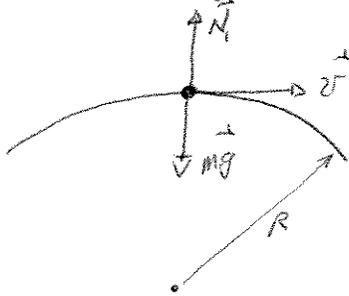
$+ 20 \text{ N} \times \sin 30^\circ)$   
 $= \underline{10.8 \text{ N}}$

2. [8 pts total] A car is driven at constant speed over a circular hill and then into a circular valley with the same radius of 40.8 m. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 81.6 kg.



- A. [4 pts] What is the speed of the car?  
 B. [4 pts] What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?

A. At the top of the first hill



Taking the direction towards the center of the circle as positive:

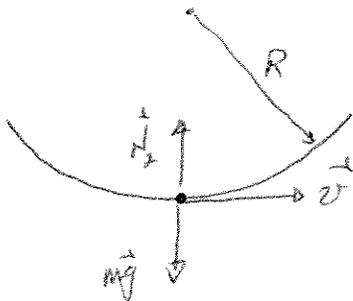
$$mg - N = m\frac{v^2}{R}$$

$$v = \sqrt{gR}$$

$$v = \sqrt{(9.8 \frac{m}{s^2})(40.8 m)}$$

$$v = 20.0 \frac{m}{s}$$

B. At the bottom of the valley



Taking the direction towards the center of the circle as positive

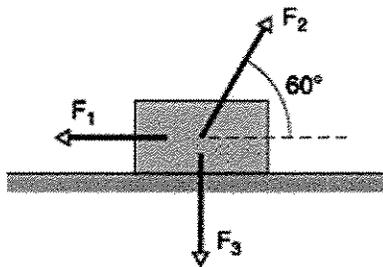
$$N_2 - mg = m\frac{v^2}{R}$$

$$N_2 = mg + m\frac{v^2}{R} = m\left(g + \frac{v^2}{R}\right)$$

$$N_2 = 81.6 \text{ kg} \left( 9.8 \frac{m}{s^2} + \frac{(20.0 \frac{m}{s})^2}{40.8 m} \right)$$

$$N_2 = 1.60 \times 10^3 \text{ N}$$

3. [12 pts total] The figure shows three forces applied to a trunk that moves to the right by 2.50 m over a frictionless floor. The force magnitudes are  $F_1 = 5.00$  N,  $F_2 = 6.00$  N, and  $F_3 = 3.00$  N.



- A. [8 pts] During the displacement, what is the net work done on the trunk by the three forces?
- B. [4 pts] If the kinetic energy of the trunk after the displacement was 20.0 J, what was the kinetic energy before the displacement?

$$A. \quad W_1 = \vec{F}_1 \cdot \vec{s} = (5.00 \text{ N}) (2.50 \text{ m}) \cos 180^\circ = -12.5 \text{ J}$$

$$W_2 = \vec{F}_2 \cdot \vec{s} = (6.00 \text{ N}) (2.50 \text{ m}) \cos 60^\circ = 7.5 \text{ J}$$

$$W_3 = \vec{F}_3 \cdot \vec{s} = (3.00 \text{ N}) (2.50 \text{ m}) \cos 90^\circ = 0$$

$$\boxed{W_{\text{TOTAL}} = -5.00 \text{ J}}$$

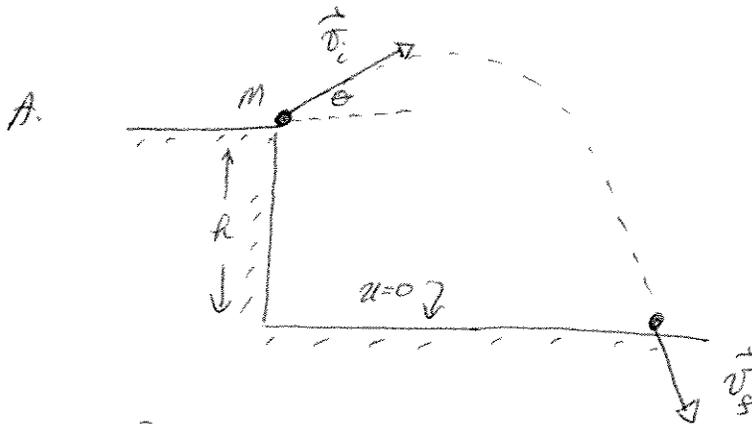
$$B. \quad K_f = K_i + W_{\text{TOTAL}}$$

$$K_i = K_f - W_{\text{TOTAL}}$$

$$= 20.0 \text{ J} - (-5.0 \text{ J})$$

$$\boxed{K_i = 25 \text{ J}}$$

4. [12 pts total] A snowball of mass 2.00 kg is fired from a cliff a height 15.3 m above level ground with an initial velocity of 10.0 m/s directed at an angle  $30^\circ$  above the horizontal.
- A. [4 pts] Using energy techniques, rather than techniques of projectile motion, find the speed of the snowball as it reaches the ground below the cliff.
- B. [4 pts] What is that speed if, instead, the launch angle is at an angle  $30^\circ$  below the horizontal?
- C. [4 pts] What is that speed if the mass is doubled?



$$m = 2.00 \text{ kg}$$

$$h = 15.3 \text{ m}$$

$$v_i = 10.0 \text{ m/s}$$

$$\theta = 30^\circ$$

$$\frac{1}{2} m v_i^2 + mgh = \frac{1}{2} m v_f^2$$

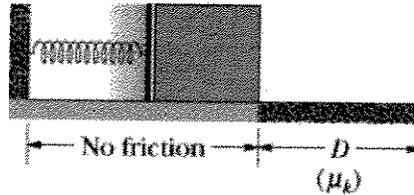
$$v_f = \sqrt{v_i^2 + 2gh} = \sqrt{(10.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(15.3 \text{ m})}$$

$$v_f = 20.0 \text{ m/s}$$

B.  $\boxed{20.0 \text{ m/s}}$  ( $\theta$  does not even enter the equation.)

C.  $\boxed{20.0 \text{ m/s}}$  ( $m$  cancels out of the equation.)

5. [12 pts total] A block of mass 4.00 kg is accelerated from rest by a compressed spring of spring constant 7.50 N/m. The block leaves the spring at the spring's relaxed length and then travels over a horizontal floor with a coefficient of kinetic friction 0.250. The frictional force stops the block in a distance of 1.50 m.



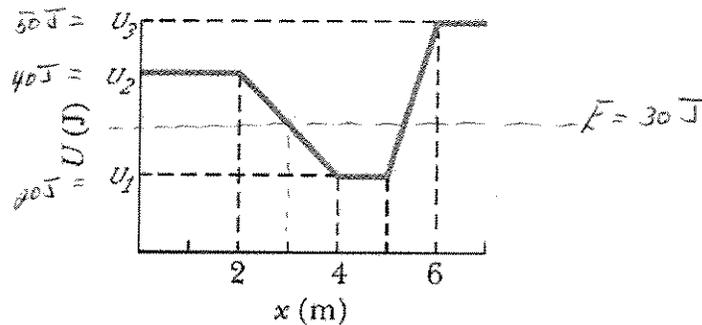
- A. [4 pts] What is the increase in the thermal energy of the block floor system?  
 B. [4 pts] What was the maximum speed of the block?  
 C. [4 pts] What was the original compression distance of the spring?

A.  $\Delta E_{Th} = \mu_k N D = \mu_k mg D = (0.250)(4.00 \text{ kg})(9.8 \text{ m/s}^2)(1.50 \text{ m})$   
 $\Delta E_{Th} = 14.7 \text{ J}$

B.  $\frac{1}{2} m v^2 = 14.7$   
 $v = \sqrt{\frac{2(14.7 \text{ J})}{4.00 \text{ kg}}} = 2.71 \text{ m/s}$

C.  $\frac{1}{2} k x^2 = \frac{1}{2} m v^2 = 14.7$   
 $x = \sqrt{\frac{2(14.7 \text{ J})}{7.50 \text{ N/m}}} = 1.98 \text{ m}$

6. [12 pts total] A particle of mass 5.00 kg can travel only along the  $x$  axis and is acted upon by a conservative force. The figure shows a plot of potential energy  $U$  versus position for the particle. In the graphs, the potential energies are  $U_1 = 20$  J,  $U_2 = 40$  J, and  $U_3 = 50$  J.



The particle is released at  $x = 4.50$  m with an initial speed of 2.00 m/s, headed in the negative  $x$  direction.

- A. [8 pts] If the particle can reach  $x = 1.00$  m, what is its speed there, and if it cannot, what is its turning point?
- B. [4 pts] What are the magnitude and direction of the force on the particle as it begins to move to the left of  $x = 4.0$  m?

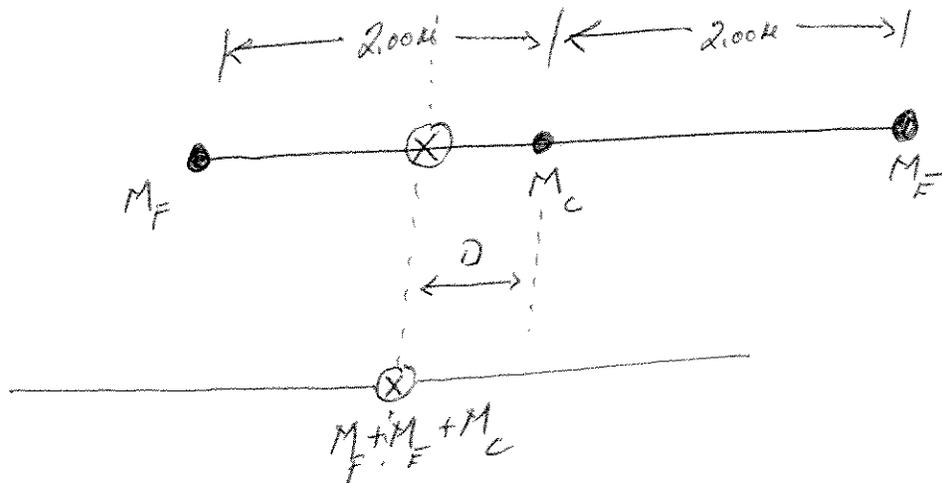
A. 
$$E = K + U = \frac{1}{2} (5.00 \text{ kg}) (2.00 \text{ m/s})^2 + 20 \text{ J} = 30.0 \text{ J}$$

Since the particle cannot have a potential energy greater than its total energy of 30 J, it cannot reach  $x = 1$  m where its potential energy would be 40 J. Since its total energy is midway between  $U_1$  and  $U_2$  its turning point is midway between 4 m and 2 m, or

3 m.

B. 
$$\bar{F}_x = - \frac{dU}{dx} = - \frac{\Delta U}{\Delta x} = - \frac{U - U_1}{2 - 4} = - \frac{40 \text{ J} - 20 \text{ J}}{-2 \text{ m}} = 10 \text{ N}$$
  
to the right.

7. [12 pts] Fred and Ethyl are enjoying Lake Merced at dusk in a canoe. Fred has a mass of 75.0 kg, Ethyl has a mass of 55.0 kg, and the canoe has a mass of 25.0 kg. The canoe is 4.00 m long. Fred and Ethyl both get up and move to the center of the boat where they sit down together. How far does the boat move?



First locate  $X_{cm}$  from the left end of the canoe (on Fred's seat)

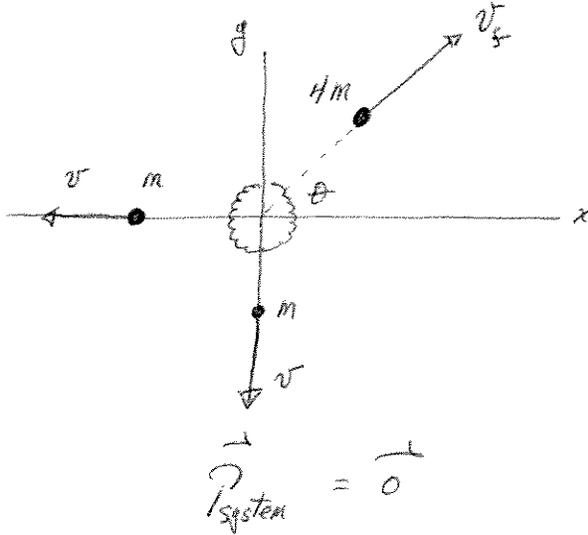
$$X_{cm} = \frac{(75.0 \text{ kg})(0) + (25.0 \text{ kg})(2.00 \text{ m}) + (55.0 \text{ kg})(4.00 \text{ m})}{75.0 \text{ kg} + 25.0 \text{ kg} + 55.0 \text{ kg}}$$

$$X_{cm} = 1.74 \text{ m}$$

The distance of the CM from the center of the canoe is  
 $2.00 \text{ m} - 1.74 \text{ m} = 0.26 \text{ m}$

After moving, Fred, Ethyl, and the center of the boat are all together at the center of mass; hence the boat moved 0.26 m

8. [12 pts] An object at rest on a frictionless table explodes, breaking into three pieces. Two of the pieces, each having mass  $m$ , fly off perpendicular to one another with the same speed of  $90 \text{ m/s}$ , one piece in the negative  $x$  direction, and the other piece in the negative  $y$  direction. The third piece has mass  $4m$ . What is the *speed and direction* of the third piece immediately after the explosion?



$$v = 90 \text{ m/s}$$

$$\textcircled{x} \quad 4m v_f \cos \theta - m v = 0 \quad \Rightarrow \quad 4 v_f \cos \theta = v$$

$$\textcircled{y} \quad 4m v_f \sin \theta - m v = 0 \quad \Rightarrow \quad 4 v_f \sin \theta = v$$

$$\frac{\textcircled{y}}{\textcircled{x}}: \quad \frac{4 v_f \sin \theta}{4 v_f \cos \theta} = \frac{v}{v}$$

$$\tan \theta = 1$$

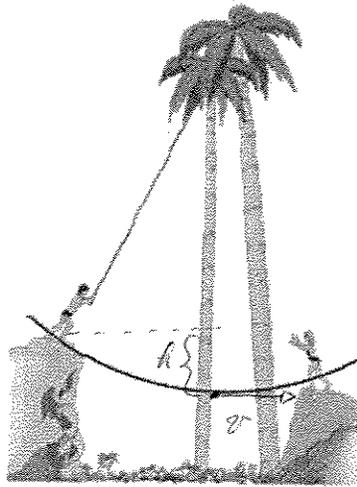
$$\theta = 45^\circ$$

$$\text{Then } v_f = \frac{v}{4 \cos \theta} = \frac{90 \text{ m/s}}{4 \cos 45^\circ}$$

$$v_f = 31.8 \text{ m/s}$$

**Extra Credit (5 points — all or nothing)**

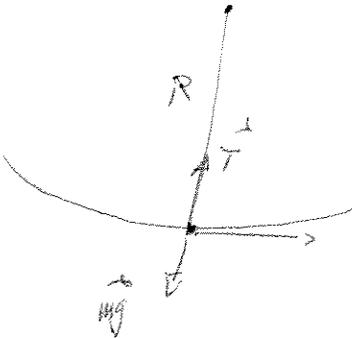
In an attempt to save Jane, Tarzan, who weighs 650 N, swings from a cliff at the end of a convenient vine that is 20 m long. From the top of the cliff to the bottom of the swing, he descends by 4 m. The vine will break if the force on it exceeds 900 N. Can Tarzan reach Jane?



The tension will be greatest at the bottom of the swing where Tarzan's speed is greatest.

$$mgh = \frac{1}{2}mv^2$$
$$v = \sqrt{2gh} = \sqrt{2(9.8 \frac{m}{s^2})(4m)}$$

$$v = 8.85 \frac{m}{s}$$



$$T - mg = m \frac{v^2}{R}$$

$$T = mg + m \frac{v^2}{R}$$

$$T = mg + \frac{mgv^2}{gR} = mg \left( 1 + \frac{v^2}{gR} \right)$$

$$T = 650N \left( 1 + \frac{(8.85 \frac{m}{s})^2}{(9.8 \frac{m}{s^2})(20m)} \right)$$

$$T = 910N > 900N \text{ so he doesn't make it.}$$