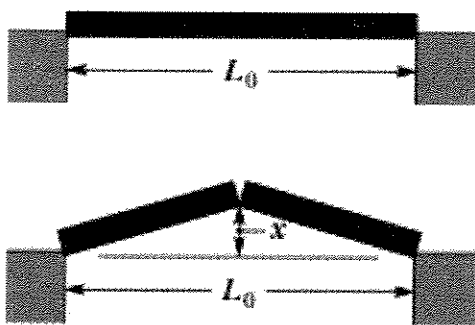


Name KEYDate 11-8-06

BE SURE TO READ THIS FIRST: Work these problems on separate sheets of paper and staple this sheet to the front. Write only on one side of each page, and box in your final answers. You must show your work or give explanations for *all* answers; I give no credit for unsupported answers. Give all answers to *no more than three* significant figures, and include appropriate units. I do give partial credit, but *only* if I can follow your work, so be as clear as possible about what you are doing.

Note: The last page contains useful physical constants and conversion factors.

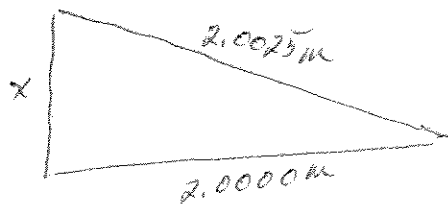
1. [8 pts] As a result of a temperature rise of 50°C , a bar with a crack at its center buckles upward. If the fixed distance L_0 is 4.0000 m and the coefficient of linear expansion of the bar is $25 \times 10^{-6} (\text{C}^\circ)^{-1}$, find the rise x of the center.



$$\Delta L = \alpha L_0 \Delta T = 25 \times 10^{-6} (\text{C}^\circ)^{-1} \times 4.00 \text{ m} \times 50 \text{ C}^\circ = 0.0050 \text{ m}$$

$$L + \Delta L = 4.0050 \text{ m}$$

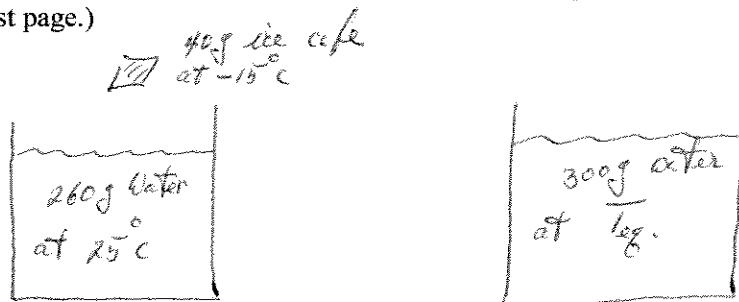
Each broken section is half this or 2.0025 m



$$x = \sqrt{(2.0025 \text{ m})^2 - (2.0000 \text{ m})^2}$$

$$x = 0.10 \text{ m}$$

2. [8 pts] A 40-g ice cube is dropped into 260 g of water in a thermally insulated container. If the water is initially at 25°C, and the ice comes directly from a freezer at -15°C, what is the final temperature at thermal equilibrium? (Neglect any heat exchanges with the glass, and note that the specific heat of ice is not the same as the specific heat of water. See the table on the last page.)



$$Q_1 = \text{Heat absorbed by ice cube to come to } 0^\circ\text{C}$$

$$= (40\text{g}) \left(0.53 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}} \right) (15^\circ\text{C}) = \underline{318\text{ cal}}$$

$$Q_2 = \text{Heat absorbed by ice in melting}$$

$$= (40\text{g}) \left(79.8 \frac{\text{cal}}{\text{g}} \right) = 3192\text{ cal}$$

$$Q_3 = \text{Heat absorbed by melted ice to come to } T_{eq}$$

$$= (40\text{g}) \left(1 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}} \right) (T_{eq} - 0) = 40 T_{eq} \text{ cal}$$

$$\left. \begin{array}{l} 318 + 3192 + 40 T_{eq} \\ (3510 + 40 T_{eq}) \end{array} \right\} \text{ (cal)}$$

$$Q_4 = \text{Heat given up by original 260g of water}$$

$$= (260\text{g}) \left(1 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}} \right) (T_{eq} - 25^\circ\text{C}) = 260 (T_{eq} - 25) \text{ cal}$$

$$= 260 T_{eq} - 6500 \text{ cal}$$

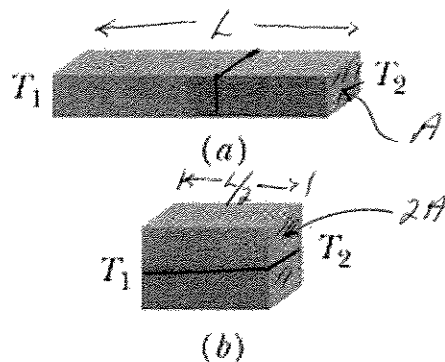
$$\sum Q = 0$$

$$3510 + 40 T_{eq} + 260 T_{eq} - 6500 = 0$$

$$300 T_{eq} = 2990$$

$$\boxed{T_{eq} = 9.97^\circ\text{C}}$$

3. [8 pts] Two identical rectangular rods of metal are welded end to end as in Figure (a), with a temperature of T_1 on the left side and a temperature of $T_2 < T_1$ on the right side. In 10 min, 100.0 J is conducted at a constant rate from the left side to the right side. How much time would be required to conduct 100 J if the rods were welded side to side as in Figure (b)? [Note: the heat flows from left to right in both cases.]

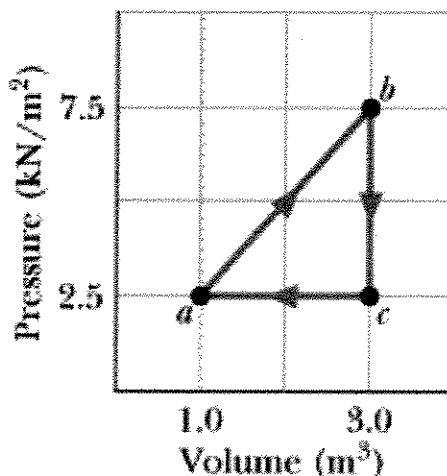


In figure (a): $\frac{dQ}{dt} = \frac{A \cdot A \cdot \Delta T}{L}$

In figure (b): $\frac{dQ'}{dt} = \frac{A (2A) \cdot \Delta T}{L/2} = 4 \cdot \frac{dQ}{dt}$

Since the heat flows 4 times as fast, it takes $\frac{1}{4}$ the time for the same 100 J to flow, or 2.5 min

4. [12 pts total] A sample of an ideal gas is taken through the cyclic process $abca$ shown in the figure. At point a , $T = 330$ K. (Note the units of pressure on the vertical axis.)



- (a) [4 pts] How many moles of gas are in the sample?
 (b) [4 pts] What is the temperature of the gas at point b ?
 (c) [4 pts] For the whole cycle, was a net amount of heat absorbed by the gas or rejected by the gas? How much?

$$(a) \quad n = \frac{P_a V_a}{R T_a} = \frac{(2.5 \times 10^3 \text{ N/m}^2) (1.0 \text{ m}^3)}{(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}) (330 \text{ K})} = \underline{0.912 \text{ mol}}$$

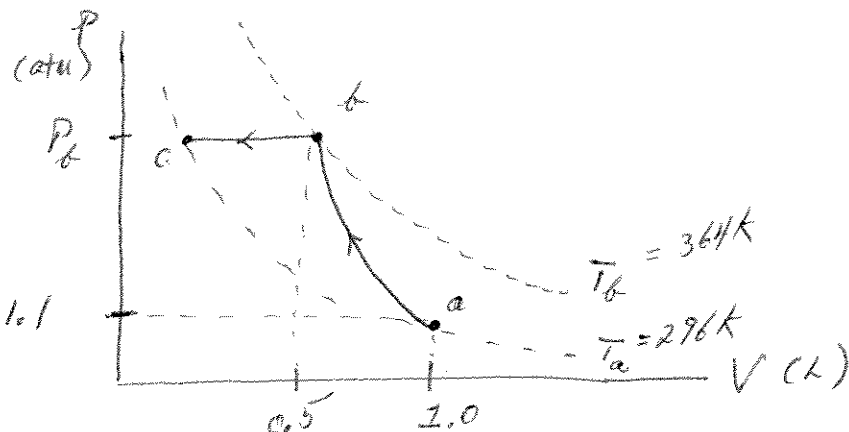
$$(b) \quad \frac{c}{f} = \frac{P_b V_b}{n R} = \frac{(7.5 \times 10^3 \text{ N/m}^2) (3.0 \text{ m}^3)}{(0.912 \text{ mol}) (8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}})} = \underline{2.97 \times 10^3 \text{ K}}$$

$$(c) \quad \text{From the 1st law of Thermodynamics: } \Delta E_{\text{cycle}} = Q_{\text{cycle}} - W_{\text{cycle}}$$

$$\begin{aligned} Q_{\text{cycle}} &= W_{\text{cycle}} = \text{Area enclosed} \\ &= \frac{1}{2} (2.0 \text{ m}^3) (5.0 \times 10^3 \text{ N/m}^2) \\ &= \underline{5.00 \times 10^3 \text{ J}} \quad (\text{absorbed}) \end{aligned}$$

5. [12 pts total] Suppose 1.00 L of a gas with $\gamma = 1.3$, initially at 296 K and 1.1 atm, is suddenly compressed adiabatically to half its initial volume.
- [4 pts] Find its final pressure.
 - [4 pts] Find its final temperature.
 - [4 pts] If the gas is cooled back to 296 K at constant pressure, what is its final volume?

$$\begin{aligned}
 (a) \quad P_a V_a^\gamma &= P_b V_b^\gamma \\
 P_b &= P_a \left(\frac{V_a}{V_b} \right)^\gamma \\
 &= (1.1 \text{ atm}) (2)^{1.3} \\
 &= \boxed{2.71 \text{ atm}}
 \end{aligned}$$



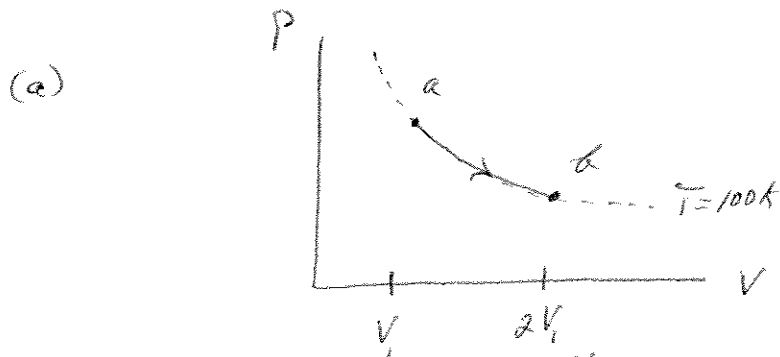
$$\begin{aligned}
 (b) \quad T_a V_a^{\gamma-1} &= T_b V_b^{\gamma-1} \\
 T_b &= T_a \left(\frac{V_a}{V_b} \right)^{\gamma-1} \\
 &= (296 \text{ K}) (2)^{0.3} \\
 &= \boxed{364 \text{ K}}
 \end{aligned}$$

(c) $PV = nRT$.
 If P is constant (as it is on the path bc) then $V \propto T$.

$$\begin{aligned}
 \frac{V_b}{T_b} &= \frac{V_c}{T_c} \\
 V_c &= V_b \cdot \frac{T_c}{T_b} = (0.50 \text{ L}) \left(\frac{296 \text{ K}}{364 \text{ K}} \right) = \boxed{0.407 \text{ L}}
 \end{aligned}$$

6. [12 pts total] Suppose 2.30 mol of an ideal gas undergoes a reversible isothermal expansion from volume V_1 to volume $V_2 = 2V_1$ at temperature $T = 100$ K.

- (a) [4 pts] Find the work done by the gas.
 (b) [4 pts] Find the entropy change of the gas.
 (c) [4 pts] If the expansion is adiabatic instead of isothermal, what is the entropy change of the gas?



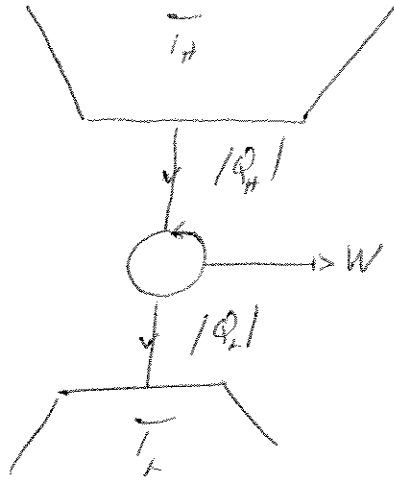
$$\begin{aligned}
 W_{at} &= \int_a^b P dV = \int_{V_a}^{V_b} \frac{nRT}{V} dV = nRT \ln \frac{V_b}{V_a} \\
 &= (2.30 \text{ mol}) (8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}) (100 \text{ K}) \ln 2 \\
 &= \boxed{1.33 \times 10^3 \text{ J}}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \Delta S &= nR \ln \frac{V_b}{V_a} + n C_V \ln \frac{T_b}{T_a} \rightarrow 0 \\
 &= (2.30 \text{ mol}) (8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}) \ln 2 \\
 &= \boxed{13.3 \text{ J/K}}
 \end{aligned}$$

(c) $\Delta S = 0$ since there would be no heat exchanged.

7. [8 pts total] A Carnot engine absorbs 64 kJ of heat and exhausts 43 kJ of heat in each cycle.
- (a) [4 pts] Calculate the engine's efficiency.
- (b) [4 pts] If the engine absorbs heat at 600°C, at what temperature does it exhaust heat?

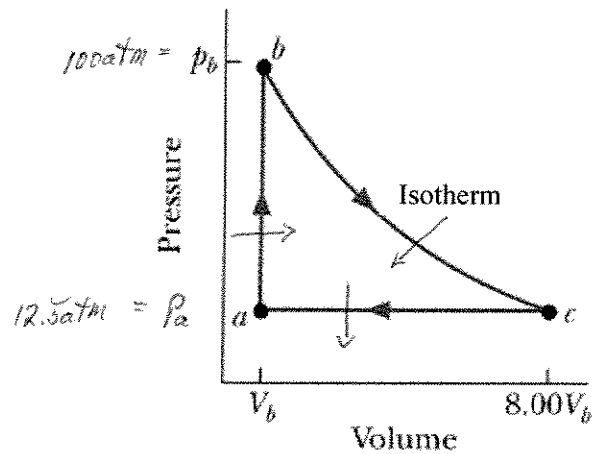


$$\begin{aligned}
 (a) \quad \epsilon &= \frac{W}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|} \\
 &= 1 - \frac{|Q_C|}{|Q_H|} \\
 &= 1 - \frac{43 \text{ kJ}}{64 \text{ kJ}} \\
 &= \underline{\underline{32.8\%}}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \epsilon_c &= 1 - \frac{T_C}{T_H} \\
 T_C &= (1 - \epsilon_c) T_H \\
 &= (1 - 0.328) \text{ (600 K)} (873 \text{ K}) \\
 &= 587 \text{ K} \\
 &= \underline{\underline{314^\circ \text{C}}}
 \end{aligned}$$

Extra Credit (10% — all or nothing)

The figure shows a reversible cycle through which 1.00 mol of a monatomic ideal gas is taken. Process bc is an *isothermal* expansion, with $P_b = 100$ atm and $V_b = 1.00$ L. Find the efficiency of the cycle.



$$\frac{1}{T_b} = \frac{1}{T_c} = \frac{P_b V_b}{nR} = \frac{(100 \text{ atm} \times 1.013 \times 10^5 \text{ Pa/atm}) (1.00 \times 10^{-3} \text{ m}^3)}{(1 \text{ mol}) (8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}})} = 1219 \text{ K}$$

$$W_{bc} = \int_{V_b}^{V_c} P dV = \int_{V_b}^{V_c} \frac{nRT}{V} dV = nRT \ln \frac{V_c}{V_b} = nRT \ln 8$$
$$= (1 \text{ mol}) (8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}) (1219 \text{ K}) \ln 8 = 21.06 \text{ kJ}$$

$$W_{ca} = P \cdot \Delta V = (12.5 \text{ atm} \times 1.013 \times 10^5 \text{ Pa/atm}) (-7 \times 10^{-3} \text{ m}^3)$$
$$= -8.86 \text{ kJ}$$

$$W_{ab} = 0$$

$$W_{\text{TOTAL}} = 12.2 \text{ kJ}$$

$$Q_{bc} = W_{bc} = 21.06 \text{ kJ}$$

$$Q_{ab} = n C_v \Delta T = (1 \text{ mol}) \left(\frac{3}{2} \times 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}} \right) \left(1219 \text{ K} - \frac{1219 \text{ K}}{8} \right) = 13.30 \text{ kJ}$$

$$Q_{in} = 34.36 \text{ kJ}$$

$$\epsilon = \frac{W_{\text{TOTAL}}}{Q_{in}} = \frac{12.2 \text{ kJ}}{34.36 \text{ kJ}} = 35.5\%$$

Useful Constants

Conversion Factors

Mechanical equivalent of heat	1 cal = 4.186 J
Pressure	1 atm = 1.013×10^5 Pa
Volume	1 liter = 10^3 cm ³ = 10^{-3} m ³
Temperature	0°C = 273 K

Universal Constants

Gas constant (R)	8.31 J/(K·mol)
Boltzmann's constant (k)	1.38×10^{-23} J/K
Stephan-Boltzmann constant (σ)	5.6703×10^{-8} W/m ² ·K ⁴
Avogadro's Number (N_A)	6.02×10^{23}

Molar Specific Heats for an Ideal Gas

Monatomic

C_V	$3R/2$
C_P	$5R/2$
γ	$5/3 = 1.67$

Diatomic

C_V	$5R/2$
C_P	$7R/2$
γ	$7/5 = 1.40$

Physical Constants for Water

Specific heat for water	1 cal/g·C°
Specific heat for ice	0.53 cal/g·C°
Heat of fusion for water	79.8 cal/g
Heat of vaporization for water	540 cal/g
Density of water	1 g/cm ³