GFauxPas
P52 Prob 6

We are given various assumptions about the nature of a theoretical PC manufacturer. We are asked to analyze the data and assumptions given in order to make inferences about the best choices for the company to make to maximize profit. We analyze these choices.

Assumptions and definitions:

The base selling price for a PC is \$950. However, we are interested in possibly lowering the price in order to increase revenue. Thus,

$price := 950 - 100 \cdot d$

$$950 - 100\, d \tag{1}$$

The current costs incurred are \$700 per computer produced. So far, \$50 000 is the monthly budget for advertising costs, but we are considering raising the advertising budget in order to increase sales. So,

$cost := 700 \cdot units + 50000 + 10000 \cdot a$

$$700\ (10000. + 100\ a) \left(1 + \frac{1}{2}\ d\right) + 50000 + 10000\ a \tag{2}$$

Baseline, 10 000 units are sold a month. We assume a decrease in price by \$100 increases units sold by 50%. We also assume that each increase in advertising budget by 10 000 will increase units sold by 200. Assuming all dependencies are linear:

$units := (1e4 + 200 \cdot a) \cdot \left(1 + \frac{1}{2} \cdot d\right)$

$$(10000. + 200\ a) \left(1 + \frac{1}{2}\ d\right) \tag{3}$$

We are told that the maximum advertising budget is \$100 000 a month, so:

$0 \le a \le 5$

$$0 \le a \textbf{ and } a \le 5 \tag{4}$$

and of course we cannot give the computers away, so the price must be positive:

$950 - 100 \cdot d > 0$

$$0 < 950 - 100\ d \tag{5}$$

Summarizing:

$units := (1e4 + 200 \cdot a) \cdot \left(1 + \frac{1}{2} \cdot d\right)$

$$(10000. + 200\ a) \left(1 + \frac{1}{2}\ d\right) \tag{6}$$

$cost := 700 \cdot units + 50000 + 10000 \cdot a$

$$700\ (10000. + 200\ a) \left(1 + \frac{1}{2}\ d\right) + 50000 + 10000\ a \tag{7}$$

$price := 950 - 100 \cdot d$

$$950 - 100\ d \tag{8}$$

$revenue := price \cdot units$

$$(950 - 100\,d)\,(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) \quad \textbf{(9)}$$

$profit := revenue - cost$

$$(950 - 100\,d)\,(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) - 700\,(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) - 50000 - 10000\,a \quad \textbf{(10)}$$

(11)

The feasible region will be defined on the *ad*-plane by $a = 0$, $a = 5$, $d < 950/100$.

$Ddprofit := diff(profit, d)$

$$-100\,(10000. + 200\,a)\left(1 + \frac{1}{2}\,d\right) + \frac{1}{2}\,(950 - 100\,d)\,(10000. + 200\,a) - 3.500000000\,10^{6} - 70000\,a \quad \textbf{(12)}$$

$Daprofit := diff(profit, a)$

$$200\,(950 - 100\,d)\left(1 + \frac{1}{2}\,d\right) - 150000 - 70000\,d \quad \textbf{(13)}$$

We find the critical points of *profit*:

$solve(\{Ddprofit = 0, Daprofit = 0\}, [a, d])$

$$[[a = -50., d = 2.265564437], [a = -50., d = -1.765564437]] \quad \textbf{(14)}$$

But these lie outside the feasible region.

We try on the curve $a = 0$

$Dda := diff(a, d)$

$$0 \quad \textbf{(15)}$$

$Daa := diff(a, a)$

$$1 \quad \textbf{(16)}$$

$s := solve(\{Ddprofit = \mathrm{lambda} \cdot Dda, Daprofit = \mathrm{lambda} \cdot Daa, a = 0\})$

$$\{a = 0., d = 0.2500000000, \lambda = 40625.\} \quad \textbf{(17)}$$

$subs\left(a = 0, d = \frac{1}{4}, profit\right)$

$$2.481250000\,10^{6} \quad \textbf{(18)}$$

(19)

This might be it.

We try on the curve $a = 5$:

lambda :='lambda'

$$\lambda \quad \textbf{(20)}$$

$solve(\{Ddprofit = \mathrm{lambda} \cdot Dda, Daprofit = \mathrm{lambda} \cdot Daa, a = 5\}, [a, d, \mathrm{lambda}])$

$$[[a = 5., d = 0.2500000000, \lambda = 40625.]] \quad \textbf{(21)}$$

$subs\left(a=5, d=\frac{1}{4}, profit\right)$

$$2.684375000\ 10^{6} \tag{22}$$

That's even better.

We'll analyze this number later.

Our best choice is then the point (*a,d*) = (5,1/4), at which the profit is ~2.684e6.
That is, one should increase the investment to the full (50 000 + 10 000 * 5) = $100 000.
Also, one should set the selling price equal to (950 - 25) = $925.

We analyze the effect of varying parameters on the model.

We had the price elasticity at 1/2. What if it were 1/4?

$units := (1e4 + 200 \cdot a)\cdot\left(1+\frac{1}{4}\cdot d\right)$

$$(10000. + 200\ a)\left(1+\frac{1}{4}\ d\right) \tag{23}$$

$cost := 700\cdot units + 50000 + 10000\cdot a$

$$700\ (10000. + 200\ a)\left(1+\frac{1}{4}\ d\right) + 50000 + 10000\ a \tag{24}$$

$price := 950 - 100\cdot d$

$$950 - 100\ d \tag{25}$$

$revenue := price\cdot units$

$$(950 - 100\ d)\ (10000. + 200\ a)\left(1+\frac{1}{4}\ d\right) \tag{26}$$

$profit := revenue - cost$

$$(950 - 100\ d)\ (10000. + 200\ a)\left(1+\frac{1}{4}\ d\right) - 700\ (10000. + 200\ a)\left(1+\frac{1}{4}\ d\right) - 50000 - 10000\ a \tag{27}$$

$Ddprofit := diff(profit, d)$

$$-100\ (10000. + 200\ a)\left(1+\frac{1}{4}\ d\right) + \frac{1}{4}\ (950 - 100\ d)\ (10000. + 200\ a) - 1.750000000\ 10^{6} - 35000\ a$$

$Daprofit := diff(profit, a)$

$$200\ (950 - 100\ d)\left(1+\frac{1}{4}\ d\right) - 150000 - 35000\ d \tag{29}$$

$Daa := diff(a, a)$

$$1 \tag{30}$$

$Dda := diff(d, a)$

$$0 \tag{31}$$

lambda :='lambda'

$$\lambda \tag{32}$$

$s := solve(\{Ddprofit = \text{lambda}\cdot Dda, Daprofit = \text{lambda}\cdot Daa, a = 5\})$

$$\{a=5.,\ d=-0.7500000000,\ \lambda=42812.50000\} \quad \textbf{(33)}$$

One can view the sensitivity of the model to the parameter as the *ratio* of the changes in the parameter to the changes in a and d.

(There are other metrics for sensitivity discussed in lecture and in the textbook. The ratio of changes seems to me to be the best one to use here.)
We first note that varying the parameter changed a not at all.

The ratio of the change in the parameter to the change in d is:

$$ratioelasticitytod := \frac{\left(\frac{1}{2}-\left(-\frac{1}{4}\right)\right)}{\left(\frac{1}{4}-\left(-\frac{3}{4}\right)\right)}$$

$$\frac{3}{4} \quad \textbf{(34)}$$

Thus, increasing the elasticity by a factor of 3/4 will decrease the price by a factor of 100 (recall $price =$ 950 - 100d).

We had the estimated effect of an additional 200 new units sold each time the advertising budget was increased by \$10 000. What if we instead estimated the effect at only 100 new units?

$$units := (1e4 + 100\cdot a)\cdot\left(1+\frac{1}{2}\cdot d\right)$$

$$(10000. + 100\,a)\left(1+\frac{1}{2}\,d\right) \quad \textbf{(35)}$$

$$cost := 700\cdot units + 50000 + 10000\cdot a$$

$$700\,(10000. + 100\,a)\left(1+\frac{1}{2}\,d\right) + 50000 + 10000\,a \quad \textbf{(36)}$$

$$price := 950 - 100\cdot d$$

$$950 - 100\,d \quad \textbf{(37)}$$

$$revenue := price\cdot units$$

$$(950-100\,d)\,(10000. + 100\,a)\left(1+\frac{1}{2}\,d\right) \quad \textbf{(38)}$$

$$profit := revenue - cost$$

$$(950-100\,d)\,(10000. + 100\,a)\left(1+\frac{1}{2}\,d\right) - 700\,(10000. + 100\,a)\left(1+\frac{1}{2}\,d\right) - 50000 - 10000\,a \quad \textbf{(39)}$$

$$Ddprofit := diff(profit, d)$$

$$-100\,(10000. + 100\,a)\left(1+\frac{1}{2}\,d\right) + \frac{1}{2}\,(950-100\,d)\,(10000. + 100\,a) - 3.500000000\,10^{6} - 35000\,a \quad \textbf{(40)}$$

$$Daprofit := diff(profit, a)$$

$$100\,(950-100\,d)\left(1+\frac{1}{2}\,d\right) - 80000 - 35000\,d \quad \textbf{(41)}$$

$Daa := diff(a, a)$

$$1 \tag{42}$$

$Dda := diff(d, a)$

$$0 \tag{43}$$

lambda :='lambda'

$$\lambda \tag{44}$$

$s := solve(\{Ddprofit = \text{lambda} \cdot Dda, Daprofit = \text{lambda} \cdot Daa, a = 5\})$

$$\{a = 5., d = 0.2500000000, \lambda = 15312.50000\} \tag{45}$$

So halving the parameter 200 results at no change at all of the optimal choices for *a* and *d.* (Clearly they change what profit would be generated at that point, but that is outside the scope of this assignment.)

Recall *lambda:*

$\lambda := 40625$

$$40625 \tag{46}$$

This is the shadow price of the units. Meaning, it is the derivative of *profit* WRT *a.*
To confirm:

$profit := (950 - 100\, d)\,(10000. + 200\, a)\left(1 + \frac{1}{2}\, d\right) - 700\,(10000. + 200\, a)\left(1 + \frac{1}{2}\, d\right) - 50000 - 10000\, a$

$$(950 - 100\, d)\,(10000. + 200\, a)\left(1 + \frac{1}{2}\, d\right) - 700\,(10000. + 200\, a)\left(1 + \frac{1}{2}\, d\right) - 50000 - 10000\, a \tag{47}$$

$subs\left(a = 5, d = \frac{1}{4}, diff(profit, a)\right)$

$$40625 \tag{48}$$

In other words, increasing profit by another \$10 000 should result in a gain of ~\$40 625.
To confirm:

$subs\left(a = 6, d = \frac{1}{4}, profit\right) - subs\left(a = 5, d = \frac{1}{4}, profit\right)$

$$40625.000 \tag{49}$$

Thus if the advertising agents are correct, the company would be well off by increasing the advertising budget by at least another \$10 000.