Solutions by FdC and AT.

1 The problem

- A cylindrical container is adiabatically insulated from the environment. An adiabatic partition divides the internal volume of the container into two chamber.
- Each chamber contains $n = 1\,\text{mol}$ of an ideal monoatomic gas, $\gamma = 5/3$.
- The volumes of the chambers are initially the same, $V_{\text{Li}} = V_{\text{Ri}}$, and the temperatures of the gases they contain are $T_{\text{Li}} = 525\,\text{K}$ and $T_{\text{Ri}} = 275\,\text{K}$, respectively.
- The partition moves until mechanical equilibrium is attained, i.e. $p_{\text{Lf}} = p_{\text{Rf}}$.

Find the final temperatures of the gases in the two chambers, T_{Lf} and T_{Rf}.

2 Relevant equations

- same initial volumes
$$V_{\text{Li}} = V_{\text{Ri}} \tag{1}$$
- same final pressures
$$p_{\text{Lf}} = p_{\text{Rf}} \tag{2}$$
- Equation of state for an ideal gas
$$pV = nRT \tag{3}$$
- Adiabatic process
$$pV^\gamma = \text{constant} \tag{4}$$
$$TV^{\gamma-1} = \text{constant} \tag{5}$$
$$p^{1-\gamma}T^\gamma = \text{constant} \tag{6}$$
$$Q = 0 \tag{7}$$
- First law of the thermodynamics
$$\Delta U = Q - L \tag{8}$$
where
$$U = \frac{3}{2}nRT \tag{9}$$

3 First attempt: the system is isolated

The pressure differential moves the partition. The initially hotter gas (left) loses internal energy (and cools down) due to work done. The initially colder gas (right) gains internal energy (and heats up) due to the "negative work". No heat is exchanged, per Eq. (7).

- Combining Eq. (2) and Eq. (3) gives
$$\frac{nRT_{\text{Lf}}}{V_{\text{Lf}}} = \frac{nRT_{\text{Rf}}}{V_{\text{Rf}}}$$
i.e.
$$\frac{V_{\text{Lf}}}{V_{\text{Rf}}} = \frac{T_{\text{Lf}}}{T_{\text{Rf}}} \tag{10}$$
- Eq. (5) for the two chambers give
$$T_{\text{Lf}}V_{\text{Lf}}^{\gamma-1} = T_{\text{Li}}V_{\text{Li}}^{\gamma-1}, \quad T_{\text{Rf}}V_{\text{Rf}}^{\gamma} = T_{\text{Ri}}V_{\text{Ri}}^{\gamma-1} \tag{11}$$
which, taking Eq. (1) into account, give
$$\frac{T_{\text{Lf}}}{T_{\text{Rf}}}\left(\frac{V_{\text{Lf}}}{V_{\text{Rf}}}\right)^{\gamma-1} = \frac{T_{\text{Li}}}{T_{\text{Ri}}}$$
Finally, plugging Eq. (11) into the last result, we get
$$\frac{T_{\text{Lf}}}{T_{\text{Rf}}} = \left(\frac{T_{\text{Li}}}{T_{\text{Ri}}}\right)^\alpha \tag{12}$$
where $\alpha = \gamma^{-1} = 3/5$.

- Since the system is isolated, the total internal energy does not change: $\Delta U = \Delta U_{\rm L} + \Delta U_{\rm R} = 0$. Since $Q_{\rm L} = Q_{\rm R} = 0$, the first law, Eq. (8), entails that $L_{\rm L} + L_{\rm R} = 0$, that is: the gas on the left performs work onto the gas on the right. Owing to Eq. (9),
$$\Delta U = 0 \quad \Rightarrow \quad \Delta T_{\rm L} + \Delta T_{\rm R} = 0$$
and hence
$$T_{\rm Lf} + T_{\rm Rf} = T_{\rm Li} + T_{\rm Ri} \tag{13}$$
- Combining Eq. (12) and Eq. (13) one gets
$$T_{\rm Rf} = \frac{T_{\rm Li} + T_{\rm Ri}}{1 + \left(\frac{T_{\rm Li}}{T_{\rm Ri}}\right)^{\alpha}} \approx 323.36\,{\rm K}, \qquad T_{\rm Lf} = \frac{T_{\rm Li} + T_{\rm Ri}}{1 + \left(\frac{T_{\rm Ri}}{T_{\rm Li}}\right)^{\alpha}} \approx 476.64\,{\rm K} \tag{14}$$

3.1 An apparent problem

Let us check that the total volume does not vary. From Eq. (11)
$$V_{\rm Lf} = V_{\rm Li}\left(\frac{T_{\rm Li}}{T_{\rm Lf}}\right)^{\beta}, \quad V_{\rm Rf} = V_{\rm Ri}\left(\frac{T_{\rm Ri}}{T_{\rm Rf}}\right)^{\beta},$$
where $\beta = (\gamma - 1)^{-1} = 3/2$. Recalling that, from Eq. (1),
$$V_{\rm Lf} + V_{\rm Rf} = \frac{V}{2}\left[\left(\frac{T_{\rm Li}}{T_{\rm Lf}}\right)^{\beta} + \left(\frac{T_{\rm Ri}}{T_{\rm Rf}}\right)^{\beta}\right]$$
the conservation of the total volume should give
$$\left(\frac{T_{\rm Li}}{T_{\rm Lf}}\right)^{\beta} + \left(\frac{T_{\rm Ri}}{T_{\rm Rf}}\right)^{\beta} = 2 \tag{15}$$
Unfortunately Eq. (14) does not seem to be compatible with Eq. (15):
$$\left(\frac{525\,{\rm K}}{476.74\,{\rm K}}\right)^{\frac{3}{2}} + \left(\frac{275\,{\rm K}}{323.36\,{\rm K}}\right)^{\frac{3}{2}} \approx 1.9403 < 2$$
This is weird. Where has the missing volume gone?

4 Second attempt: the total volume should not change

We want to enforce the conservation of the volume
$$V_{\rm Lf} + V_{\rm Rf} = V_{\rm Li} + V_{\rm Ri} \tag{16}$$
By plugging Eq. (1) and Eq. (10) into the previous one we get
$$V_{\rm Rf}\left(1 + \frac{T_{\rm Lf}}{T_{\rm Rf}}\right) = 2V_{\rm Ri}$$
Now, from the second of Eq. (11)
$$V_{\rm Ri} = V_{\rm Rf}\left(\frac{T_{\rm Rf}}{T_{\rm Ri}}\right)^{\beta}$$
After plugging this result into the previous one and discarding the common factor $V_{\rm Rf}$ we get
$$1 + \frac{T_{\rm Lf}}{T_{\rm Rf}} = 2\left(\frac{T_{\rm Rf}}{T_{\rm Ri}}\right)^{\beta}$$
The ratio of the initial temperatures in the lhs of this equation can be taken from Eq. (12), so that
$$1 + \left(\frac{T_{\rm Li}}{T_{\rm Ri}}\right)^{\alpha} = 2\left(\frac{T_{\rm Rf}}{T_{\rm Ri}}\right)^{\beta}$$
Therefore we get
$$T_{\rm Rf} = T_{\rm Ri}\left[\frac{1 + \left(\frac{T_{\rm Li}}{T_{\rm Ri}}\right)^{\alpha}}{2}\right]^{\frac{1}{\beta}} \approx 316.89\,{\rm K}, \qquad T_{\rm Lf} = T_{\rm Li}\left[\frac{1 + \left(\frac{T_{\rm Ri}}{T_{\rm Li}}\right)^{\alpha}}{2}\right]^{\frac{1}{\beta}} \approx 467.10\,{\rm K} \tag{17}$$

4.1 An apparent problem

Let us evaluate the changes in internal energy of the gases in the two chambers. Since the processes are adiabatic, these changes also equal the work exchanged by the gases:

$$\Delta U_{\mathrm{L}} = -L_{\mathrm{L}} = nR(T_{\mathrm{Lf}} - T_{\mathrm{Li}}) \approx -481.17\,\mathrm{J}, \qquad \Delta U_{\mathrm{R}} = -L_{\mathrm{R}} = nR(T_{\mathrm{Rf}} - T_{\mathrm{Ri}}) \approx 348.13\,\mathrm{J} \tag{18}$$

Therefore

$$\Delta U = -L = -133.04\,\mathrm{J} \tag{19}$$

Where did the missing internal energy go? How was the missing work done?

I would expect that the pressure differential does all the work, with no need for an exchange of energy, work or heat with the environment.