

$$\begin{aligned}
E\left(\frac{r-1}{X-1}\right) &= \sum_{x=r}^{\infty} \frac{r-1}{x-1} \binom{x-1}{r-1} p^r (1-p)^{x-r} \\
&= \sum_{x=r}^{\infty} \frac{r-1}{x-1} \frac{(x-1)!}{(r-1)!(x-r)!} p^r (1-p)^{x-r} \\
&= \sum_{x=r}^{\infty} \frac{(x-2)!}{(r-2)!(x-r)!} p^r (1-p)^{x-r} \\
&= \sum_{x=r}^{\infty} \binom{x-2}{r-2} p^r (1-p)^{x-r} \\
&= \sum_{y=r-1}^{\infty} \frac{1}{r-2} \binom{y-1}{r-1} p^r (1-p)^{y+1-r} \\
&= \frac{(1-p)}{r-2} \sum_{y=r-1}^{\infty} \binom{y-1}{r-1} p^r (1-p)^{y-r}
\end{aligned}$$