coordinate is involved in this special motion. In general there will be N values of ω if the matrices A and B are $N \times N$ and these values are known as *normal frequencies* or *eigenfrequencies*.

Putting (9.8) into (9.7) yields

$$-\omega^2 \mathsf{A}\mathsf{x} + \mathsf{B}\mathsf{x} = (\mathsf{B} - \omega^2 \mathsf{A})\mathsf{x} = \mathsf{0}. \tag{9.9}$$

Our work in section 8.18 showed that this can have non-trivial solutions only if

$$|\mathsf{B} - \omega^2 \mathsf{A}| = 0. \tag{9.10}$$

This is a form of characteristic equation for B, except that the unit matrix I has been replaced by A. It has the more familiar form if a choice of coordinates is made in which the kinetic energy T is a simple sum of squared terms, i.e. it has been diagonalised, and the scale of the new coordinates is then chosen to make each diagonal element unity.

However, even in the present case, (9.10) can be solved to yield ω_k^2 for $k = 1, 2, \ldots, N$, where N is the order of A and B. The values of ω_k can be used with (9.9) to find the corresponding column vector x^k and the initial (stationary) physical configuration that, on release, will execute motion with period $2\pi/\omega_k$.

In equation (8.76) we showed that the eigenvectors of a real symmetric matrix were, except in the case of degeneracy of the eigenvalues, mutually orthogonal. In the present situation an analogous, but not identical, result holds. It is shown in section 9.3 that if x^1 and x^2 are two eigenvectors satisfying (9.9) for different values of ω^2 then they are orthogonal in the sense that

$$(\mathsf{x}^2)^{\mathrm{T}} \mathsf{A} \mathsf{x}^1 = 0 \qquad \text{and} \qquad (\mathsf{x}^2)^{\mathrm{T}} \mathsf{B} \mathsf{x}^1 = 0.$$

The direct 'scalar product' $(\mathsf{x}^2)^{\mathrm{T}}\mathsf{x}^1$, formally equal to $(\mathsf{x}^2)^{\mathrm{T}}\mathsf{I}\,\mathsf{x}^1$, is not, in general, equal to zero.

Returning to the suspended rod, we find from (9.10)

$$\left| \frac{Mlg}{12}\begin{pmatrix} 6 & 0 \\ 0 & 3 \end{pmatrix} - \frac{\omega^2 Ml^2}{12}\begin{pmatrix} 6 & 3 \\ 3 & 2 \end{pmatrix} \right| = 0.$$

Writing $\omega^2 l/g = \lambda$, this becomes

$$\begin{vmatrix} 6 - 6\lambda & -3\lambda \\ -3\lambda & 3 - 2\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda^2 - 10\lambda + 6 = 0,$$

which has roots $\lambda = 5 \pm \sqrt{19}$. Thus we find that the two normal frequencies are given by $\omega_1 = (0.641g/l)^{1/2}$ and $\omega_2 = (9.359g/l)^{1/2}$. Putting the lower of the two values for ω^2, namely $(5 - \sqrt{19})g/l$, into (9.9) shows that for this mode

$$x_1 : x_2 = 3(5 - \sqrt{19}) : 6(\sqrt{19} - 4) = 1.923 : 2.153.$$

This corresponds to the case where the rod and string are almost straight out, i.e. they almost form a simple pendulum. Similarly it may be shown that the higher