Ch 2. #7)

Show that $\sum_{k=1}^{n} k^p = \frac{n^{p+1}}{p+1} + An^p + Bn^{p-1} + Cn^{p-2} + \cdots$.

This formula holds for $p=0$

$$\sum_{k=1}^{n} k^0 = \frac{n^1}{1}$$

Now suppose the formula holds $\forall n \{n \in \mathbb{N} \mid n \leq p\}$

It follows that $\sum_{k=1}^{n} k^p = \frac{n^{p+1}}{p+1} + An^p + Bn^{p-1} + Cn^{p-2} + \cdots,$

and $\sum_{k=1}^{n} k^{p-1} = \frac{n^p}{p} + An^{p-1} + Bn^{p-2} + Cn^{p-3} + \cdots,$

$\vdots$

$$\sum_{k=1}^{n} k^0 = \frac{n^1}{1}$$

Then

$$\sum_{k=1}^{n} k^{p+1} = \frac{n^{p+2}}{p+2} + An^{p+1} + Bn^p + Cn^{p-1} + \cdots$$