

Since k^{p+1} is a polynomial we can use the method from problem 6 to derive the sum formula of $\sum_{k=1}^n k^{p+1}$ and ultimately re-express it in the form we want.

$$(1+k)^{p+2} - k^{p+2} = \sum_{n=0}^{p+2} \binom{p+2}{n} k^n - k^{p+2} \quad [\text{Binomial Theorem}]$$

$$= \sum_{n=0}^{p+1} \binom{p+2}{n} k^n + \binom{p+2}{p+2} k^{p+2} - k^{p+2}$$

$$= \sum_{n=0}^{p+1} \binom{p+2}{n} k^n$$

Let $k=1, \dots, r$ then sum the list of r equations.

$$k=1 \rightarrow (2)^{p+2} - (1)^{p+2} = \sum_{n=0}^{p+1} \binom{p+2}{n} 1^n$$

$$k=2 \rightarrow (3)^{p+2} - (2)^{p+2} = \sum_{n=0}^{p+1} \binom{p+2}{n} 2^n$$

\vdots

$$k=r \rightarrow (1+r)^{p+2} - (r)^{p+2} = \sum_{n=0}^{p+1} \binom{p+2}{n} r^n$$