$$\binom{p+2}{p+1}\sum_{k=1} k^{p+1} = \binom{p+2}{p+2} r^{p+2} + \binom{p+2}{p+1} r^{p+1} + \sum_{j=0} \binom{p+2}{j} r^j - 1 - \sum_{j=0} \sum_{k=1} \binom{p+2}{j} k^j$$

$$= \binom{p+2}{p+2} r^{p+2} + \binom{p+2}{p+1} r^{p+1} + \sum_{j=0}^{p} \binom{p+2}{j} r^j - 1 - \sum_{j=0}^{p} \sum_{k=1}^{r-1} \binom{p+2}{j} k^j - \sum_{j=0}^{p} \binom{p+2}{j} r^j$$

$$= \binom{p+2}{p+2} r^{p+2} + \binom{p+2}{p+1} r^{p+1} - 1 - \sum_{j=0}^{p} \sum_{k=1}^{r-1} \binom{p+2}{j} k^j$$

Rewriting both sides of the equation we have,

$$(p+2)\sum_{k=1}^{r} k^{p+1} = r^{p+2} + (p+2) r^{p+1} - 1 - \sum_{j=0}^{p} \sum_{k=1}^{r-1} \binom{p+2}{j} k^j$$

$$\Rightarrow \sum_{k=1}^{r} k^{p+1} = \frac{r^{p+2}}{p+2} + r^{p+1} - \frac{1}{p+2}\left(\sum_{j=0}^{p} \sum_{k=1}^{r-1} \binom{p+2}{j} k^j + 1 \right)$$