Parallel Transport with Respect to Inertial Frames

Parallel transport is defined by the fact that covariant derivative of the vector as it moves along the curve should be zero .

Now the covariant derivative of a tensor has two parts :
1)The ordinary derivative.
2)The part containing the "affine connection".
The affine connection is zero in all local inertial frames while it may be non zero innon-inertial frames.

Covariant derivative along a curve $x^{\mu}(\tau)$, for an arbitrary frame

$$\frac{dA^{\mu}}{d\tau} + \Gamma^{\mu}{}_{\nu\lambda} \frac{dx^{\lambda}}{d\tau} A^{\nu} = 0$$

For an inertial frame:

$$\frac{dA^{\mu}}{d\tau} = 0$$

Since

$$\Gamma^{\mu}{}_{\nu\lambda} = 0$$

For inertial framrs.

We connect the end points of the path of parallel transport by a chain of "local inertial points"

Along the path, we have

$$\frac{dA^{\mu}}{d\tau} = 0$$

Therefore the components of the tensor remain constant as we move along the "chain of local inertial points"

So if we parallel transport a vector along different paths **starting from the same point** the components do not change, when referred to the local inertial frames.

[NB: If the components of a tensor are zero in any frame they are also zero in all other frames.
=> If the covariant derivative=0 in any frame it should be zero in all other frames, inertial or non-inertial]