The EFT literature that I have seen,typically says that when going to the EFT the heavy particle is "integrated out".
Does it mean that now it can only appear as a propagator ?
And that "integrating out" means to solve the functional integral for some field,in the generating functional and is it equivalent to applying the Wick's Theorem to generate the propagators in the canonical formalism ?
So does it mean that "integrating out" a field is not unique to an EFT,rather it is an essential part of solving a scattering amplitude ?

At this point they also introduce the operator product expansion.

$$\mathcal{O}(x)\mathcal{O}(y) = \Sigma_i \mathcal{C}_i(x,y)\mathcal{O}_i(x)$$

under the condition that $x \to y$.In the context of scattering amplitudes these operators are the currents at the vertices so does the condition $x \to y$ imply that this expansion is only valid in the UV limit of the scattering process ?
Furthermore,in general,is OPE an essential part of an EFT ?

Regarding the feynman diagrams,for example considering a quark decay at lowest order,the full theory diagram has two vertices and a W propagator.The effective theory diagram will have the four external quark lines all at one vertex with an effective coupling given (at leadng order ?) as $G_F = \frac{g^2}{m^2}$.It seems this is because in the heavy propagator :

$$D_{\mu\nu}(x,y) = \frac{g_{\mu\nu}}{\Box + m^2} \int d^4p e^{ip(x-y)}$$

The denominator can be expanded out as $\frac{1}{\Box + m^2} = \frac{1}{m^2}(1 - \frac{\Box}{m^2} + ...)$ but it is the position space delta function : $\int d^4p e^{ip(x-y)}$ which forces the current operators to be at the same point in the Dyson series term :

$$g^2 \int d^4x d^4y \psi(\bar{x})\gamma^\mu \psi(x) D_{\mu\nu}(x,y)\psi(\bar{y})\gamma^\nu \psi(y)$$

I am confused that if an EFT is an approximation to the full theory and its interaction vertex does not respect the gauge symmetry of the full theory so how come the delta function is present in the expression of the propagator ?
Infact the only approximation taken (which gives the effective coupling G_F) is from the expansion of the denominator $\frac{1}{\Box + m^2} = \frac{1}{m^2}(1 - \frac{\Box}{m^2} + ...)$.The delta function should come about naturally from writing the propagator in the above form.