

$$\begin{aligned}
\dot{Q}_x - \dot{Q}_{x+dx} - \dot{Q}_{conv} + \dot{\phi}''' &= 0 \\
-\lambda(\delta W)\frac{dT}{dx} + \lambda(\delta W)\frac{dT}{dx} + \lambda(\delta W)\frac{d^2T}{dx^2}dx - \alpha(dx.W)(T - T_\infty) + \dot{\phi}'''(W.dx.\delta) &= 0 \\
\frac{d^2T}{dx^2} - \frac{\alpha}{\lambda\delta} T + \frac{\alpha a}{\lambda\delta} \sin(kx) + \frac{\dot{\phi}'''}{\lambda} &= 0 \\
m^2 = \frac{\alpha}{\lambda\delta}, n = \frac{\alpha a}{\lambda\delta}, p = \frac{\dot{\phi}'''}{\lambda} \\
\frac{d^2T}{dx^2} - m^2 T + n\sin(kx) &= -n \sin(kx) - \frac{\dot{\phi}'''}{\lambda}
\end{aligned}$$

Homogeneous solution:

$$T_H = Ae^{mx} + Be^{-mx}$$

Particular solution:

$$\begin{aligned}
T_p &= C_1 + C_2\sin(kx) + C_3\cos(kx) \\
-C_2k^2 \sin(kx) - C_3k^2 \cos(kx) - m^2C_1 - C_2m^2\sin(kx) - m^2C_3\cos(kx) &= -n\sin(kx) - p
\end{aligned}$$

To find the C_1, C_2 , and C_3 , the coefficients at the l.h.s. must be the same with the coefficients at the r.h.s. To determine A and B in the homogeneous solution, two boundary conditions are used: First at $x = 0$:

$$-\lambda \frac{dT}{dx} = 0$$

Second at $x = L$

$$-\lambda \frac{dT}{dx} = 0$$