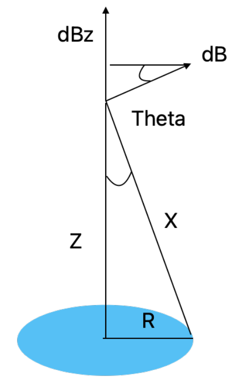
Part a.



Known:

z, B_{axis}, B_{center}

Seek:

I, r

First, the change in magnetic field will not be equal to the magnetic field at the axis because the magnetic field will act on the point in a different manner at each point, so we need to solve for the magnetic field at the axis.

$$dB = \frac{\mu_0 I}{4\pi x^2} dl \times \hat{r}$$

$$dB_z = dB_{axis}$$

$$dB_{axis} = dB sin(\theta)$$

$$dB_{axis} = \frac{\mu_0 I\ dl}{4\pi x^2} sin(\theta)$$

Now to simplify using Pythagoreans Theorem.

$$x^2 = z^2 + r^2$$

$$dB_{axis} = \frac{\mu_0 I\ dl}{4\pi (z^2 + r^2)^2} sin(\theta)$$

Now to solve for the sin with our trig identities.

$$sin(\theta) = \frac{r}{x}$$

$$x = \sqrt{z^2 + r^2}$$

$$sin(\theta) = \frac{r}{\sqrt{z^2 + r^2}}$$

And now we can plug it back in.

$$dB_{axis} = \frac{\mu_0 I\ dl}{4\pi (z^2 + r^2)^2} * \frac{r}{\sqrt{z^2 + r^2}}$$

$$dB_{axis} = \frac{\mu_0 I r\ dl}{4\pi (z^2 + r^2)^{3/2}}$$

Let’s integrate.

$$B_{axis} = \frac{\mu_0 I r}{4\pi(z^2 + r^2)^{3/2)}} \int dl$$

$$B_{axis} = \frac{2\mu_0 \pi I r^2}{4\pi(z^2 + r^2)^{3/2}}$$

$$B_{axis} = \frac{\mu_0 I r^2}{2(z^2 + r^2)^{3/2}}$$

Now that the axis magnetic field is expressed, lets find an equation for the magnetic field at the center.

$$dB_{center} = \frac{\mu_0 I}{4\pi r^2} * dl$$

$$B_{center} = \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$B_{center} = \frac{2\mu_0 \pi I r}{4\pi r^2}$$

$$B_{center} = \frac{\mu_0 I r}{2r}$$

Now we can rearrange to use I to find r.

$$I = \frac{2B_{center} r}{\mu_0}$$

We can now plug that back into our expression for the field at the axis.

$$B_{axis} = \frac{2\pi r^2 * \mu_0 (\frac{2B_{center} r}{\mu_0})}{4\pi(z^2 + r^2)^{3/2}}$$

$$B_{axis} = \frac{B_{center} r^3}{(z^2 + r^2)^{3/2}}$$

Now here's where I'm stuck because even if I move the central magnetic field value to the left, I still can't isolate the r because it's trapped in the square root.

$$\frac{B_{axis}}{B_{center}} = \frac{r^3}{(z^2 + r^2)^{3/2}}$$

I tried squaring both sides maybe to try to escape the toils of that cursed square root.

$$\frac{B_{axis}^2}{B_{center}^2} = \frac{r^6}{(z^2 + r^2)^3}$$

But if you expand this out then it just really get's ugly. I'm not sure where to go from here, any tips would be greatly appreciated.

Part b.

My idea here was to use whatever value I get for r and plug that back into the expression for the magnetic field at the center.