Effect of Varying Surface Areas of Cardboard Cutout Circles on Damping Coefficient of a Pendulum

Introduction

A pendulum is a universally understood and utilized instrument in the present day with applications that range from comprehending fundamental concepts such as simple harmonic motion, gravity, and energy conservation to timekeeping as a device within mechanical clocks. However, what interested me in particular was the fact that upon learning about the fundamental behaviors of pendulums within concepts such as simple harmonic motion, was that despite their theoretical background, pendulums as they are applied in real-world scenarios including the back-and-forth movement of a swing do not go on forever. Swings are classic real-world examples of a pendulum and it is a known fact that swings require constant force being applied in order to continue oscillating. As a result, I came across the phenomenon of damping and realized that there was little-to-no information or data on the relationship between surface area and the measure of damping (damping coefficient). Although this investigation is primarily focused on understanding the physical principles of damping and its relationship to surface area, it has various real-world applications. Most notably is that there exists a connection between energy efficiency and its impact on global energy consumption; if we are able to minimize the energy losses due to friction and air resistance, it can contribute to optimizing energy-efficient designs in various industries, and thereby encourage greater sustainable energy usage. Thus, I aim to explore this concept as an individual experiment of my choice through a method that I came about on my own.

Research Question: To what extent does the surface area (cm^2) attached to a pendulum bob have an affect on the damping coefficient?

Background:

Simple Pendulum Model

Pendulums are defined as objects that swing back and forth about a fixed point as a result of gravity. A simple pendulum is a theoretical model which consists of a point mass that is suspended from a string of negligible mass with length, l, which is further assumed to be rigid and inelastic. Once the point mass is displaced from its equilibrium position (displacement, $s = 0$) as shown in Figure 1[1], the pendulum begins to oscillate back and forth with a measurable and specific natural frequency, f, which is a frequency that a pendulum tends to oscillate in when driving forces are negligible. Simple pendulums are common representations of simple harmonic motion (SHM). Simple harmonic motion states that a restoring force has a magnitude proportional to the displacement and where the acceleration is also proportional to the displacement. The time period of oscillation of a simple pendulum is given by the following formula:

$T = 2\pi\sqrt{(l/g)}$

1

where T is the time period, l is the length of the string of the pendulum, and g is the acceleration due to gravity.

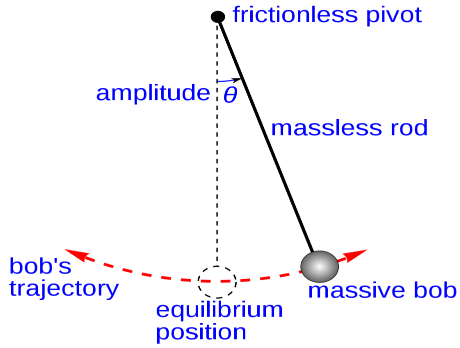


Figure 1. Labeled diagram of a simple harmonic pendulum system.

Damping

Within the confines of simple harmonic motion where the restoring force is proportional to the displacement of the mass from equilibrium and always acts in the opposite direction to the displacement, the damping forces are assumed to be non-existent as there is no dissipation of energy and the amplitude does not decrease over time. However, in real-world pendulum systems, there are almost always dissipative forces present such as friction and/or air resistance, which can lead to the damping of the system, and thus, the gradual decrease in the amplitude of the oscillations (LumenLearning). In these cases, the motion is no longer following the theoretical simple harmonic model, and the damping coefficient must be accounted for to understand and analyze the various behaviors of the system. In terms of the background of this experiment, there is a direct correlation between attached surface area and damping since there is a proportional relationship between surface area and the extent to which damping affects a system. The reason for this is because as a pendulum oscillates, it displaces air molecules it encounters in its trajectory due to its motion. As a result, a region of higher pressure may be observed in front of the pendulum bob and a region of lower pressure can be observed behind the pendulum bob (WondriumDaily). Due to this difference in pressure, the air molecules in the higher-pressure region flow towards the lower-pressure region, thereby generating a drag force on the pendulum's motion (WondriumDaily). This is due to the fact that when an object moves through a medium it experiences a resistive force, known as fluid friction. In order to measure the damping effect on the pendulum, it is pivotal to consider the effects of friction and other dissipative forces that act on the pendulum as it oscillates. In order to quantify and be able to analyze the effect that these dissipative forces have on a pendulum's system, the damping coefficient of the system can be calculated and observed under different circumstances. The damping coefficient, b, is a measure of the rate at which the amplitude decreases due to damping, and can be further explained to be the ratio between the damping force and velocity of the oscillating object (Alonso and Finn). Generally, the damping force is proportional to the velocity of the object and acts in the opposite direction to the motion; therefore, as the object oscillates, its amplitude steadily decreases. This can be denoted with the following equation:

$$F = - b\frac{dx}{dt},$$

where b is the damping coefficient, x is the displacement of the mass from its equilibrium position, and $\frac{dx}{dt}$ refers to the change in displacement with respect to time where t can also be referred to as instantaneous velocity (Alonso and Finn).

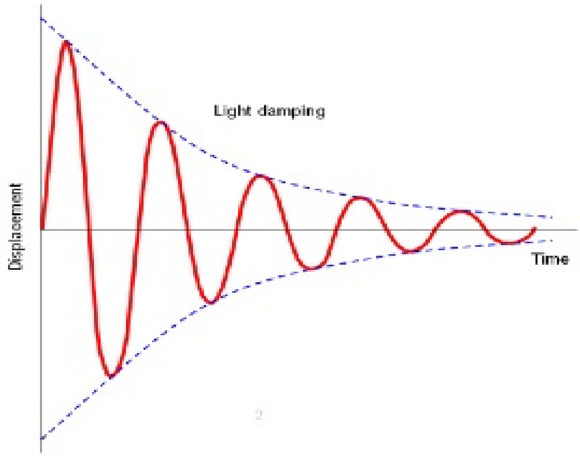


Figure 2. Graphical representation of the light damping that occurs in simple harmonic systems.

<u>Logarithmic Decrement Equation</u>

The logarithmic decrement equation is an important and widely used method to determine the damping coefficient in mechanical systems. It assumes that the amplitude of an oscillating system steadily decreases with time. The equation measures the damping coefficient of a simple pendulum by measuring the time periods of the pendulum for two consecutive oscillations and the number of oscillations between them. In order to derive the equation, we must consider the motion of the pendulum as a damped harmonic oscillator; hence, the differential equation of motion for a damped harmonic oscillator is utilized (Alonso and Finn):

$$\frac{d^2x}{dt^2} + 2\zeta\omega n\left(\frac{dx}{dy}\right) + \omega n^2 x = 0,$$

where x is the displacement of the mass from equilibrium, t is time, ωn is the natural frequency of the oscillator, and ζ is the damping ratio. However, since the solution to this differential equation is outside the scope of the Physics HL course, I will omit the working out and include the conclusion that M. Alonso and E. J. Finn reached in "Chapter 11: Damping Oscillations," in Physics, Addison-Wesley Publishing Company, 1992, pp. 401-407:

$$b = \frac{ln\left(A_{n+m}/A_n\right)}{mT},$$

where b is the damping coefficient, A_n is the amplitude after a fixed number of oscillations "n", A_{n+m} is the second amplitude after a fixed number of oscillations "m", m is the number of complete cycles between the two successive amplitudes A_n and A_{n+m}, and T is the time taken for m cycles to occur between the two amplitudes A_n and A_{n+m}.

Hypothesis:

Based on the background research I have conducted, I hypothesize that as the surface area increases, there will be a greater damping force experienced by the pendulum. Therefore, the magnitude of the damping coefficient, which is a measure of the amount of damping in a system, will increase. This is due to the fact that a larger surface area creates more contact with its surrounding air, leading to increased air resistance and thus higher damping since the energy of the pendulum's swing is dissipated in the air (Dahmen).

Experiment

Methodology

Variables	**Measurement of Variables**	**Range of Measurements**
Independent: Surface area attached to pendulum bob (cm^2) The surface area of the cardboard attached to the pendulum bob will be varied to	The surface area will be measured precisely through the use of a laser cutter that ensures that correct radii have been cut, and thus, the accuracy and precision in the variation of the surface areas may be maximized. .	There will be a total of seven varying surface areas cut using the laser cutter with diameter measurements of 0.0252 m, 0.0302 m, 0.0352 m, 0.0402 m, 0.0452 m, 0.0502 m, and 0.0552 m and corresponding measurements of 4.988 cm^2, 7.163 cm^2, 9.731 cm^2, 12.692 cm^2, 14.200 cm^2, 19.792 cm^2, and 23.931 cm^2, respectively. The large data range that can be collected for the experiment will ensure that clear trends may be identified on the effect of surface area on the damping coefficient.
Dependent: 1. Damping coefficient (b) 2. Amplitudes (m) after nth and nth + mth oscillation (where n=8 and m = 3) 3. Time (s) between nth and mth oscillations	1. The damping coefficient, measured as a dimensionless quantity in the context of the logarithmic decrement equation, will be calculated using the aforementioned equation once necessary values are obtained for the nth and mth amplitudes, as	1. The range of the damping coefficient's values will be based on the varying surface areas attached to the pendulum; therefore, there will be seven separate and distinct values for the damping coefficient. 2. Similarly for the amplitudes and time between the nth and mth oscillations, there will also be seven separate and distinct values

(where n=8 and m = 3)	well as the time between the oscillations. 2. As a result of the varying surface areas, the amplitudes of the nth and mth oscillations will differ due to varying degrees of air resistance. 3. As an added effect, the time between the nth and mth oscillation will also differ due to its similar interaction with the varying degrees of air resistance between each attached surface area on the pendulum bob.	corresponding to each aspect of the logarithmic decrement equation.

Control Variables	**Method for Control**	**Justification for Control**
1. Length of string 2. Mass of pendulum bob 3. Initial amplitude of displacement 4. Pressure and temperature of surroundings	1. The same string will be used throughout the entirety of the experiment of a known length of 0.3336 $\pm$ 0.002 m. 2. The same pendulum bob will be used throughout the entirety of the experiment of a known mass of 71 grams. 3. The initial amplitude of displacement has been measured to be 0.007 $\pm$ 0.001 m from its equilibrium position and will be kept constant throughout different trials of the experiment.	1. There are various reasons for why the length of the string must be held constant. Namely, the time period of the pendulum is directly affected by the length of the string. Thus, in order to maintain consistency between the experiments and trials, the length of the string must be kept constant or there will be incoherent results as it would not be known whether or not the damping effect on the system is increasing or decreasing due to the length of the string. 2. The mass of the pendulum bob must be kept constant in order to ensure that

	4. The experiment will be conducted in the same area in order to approximate that the pressure and temperature will be the same throughout different trials of the experiment.	experimental conditions are consistent and the variables being measured are being affected only by the independent variable. 3. The initial amplitude is arguably the most important variable to be kept constant as it affects the amplitude reached by a pendulum. Therefore, experimental conditions associated with it must be held constant in order to avoid discrepancies in the data found between measured values for different surface areas attached to the pendulum bob. 4. The pressure and temperature of the surroundings affect the frequency of collisions that the attached cardboard cutout of a circle on the pendulum bob has with its surrounding air molecules. As a result, the experimental conditions will not be consistent and will be reflected in the results of the value of the varying damping coefficients.

Materials

- Retort stand (1)
- Pendulum bob (constant with mass of 71 grams) with a hook
- G-clamp (1)
- Retort clamp (1)
- Cross clamp (1)
- Thin thread (constant with length of 0.3336 ± 0.002 m (1)
- 0.3 meter ruler (1)
- 6 cardboard cutouts of circles of varying surface areas (6)

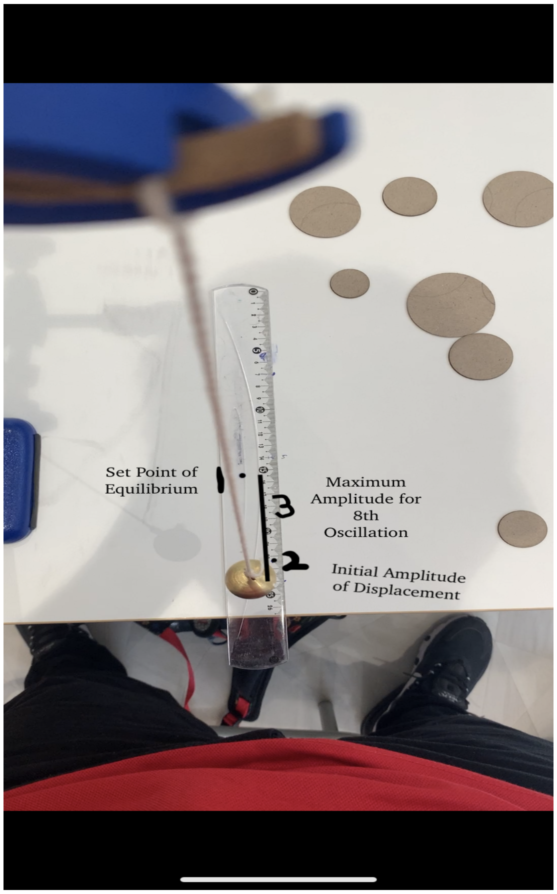
- Laser cutter (1)
- Super glue (1)
- Micrometer screw gauge (1)

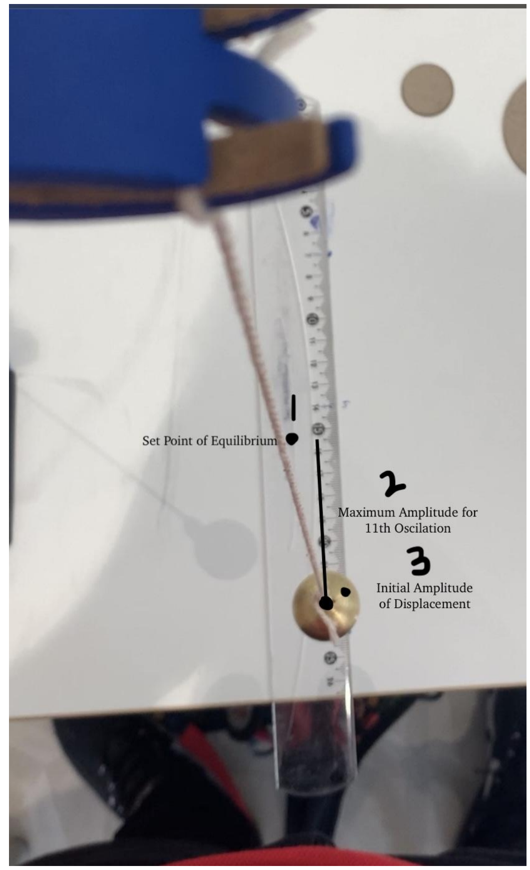
<u>Evolution of plan</u>

A preliminary experiment had been conducted in order to identify the most appropriate and consistent method to measure the maximum amplitude of a pendulum. Initially a motion sensor was utilized; however, it was sensitive to motion and at times produced nonsensical results. Next, a photogate was used to measure the varying maximum amplitudes; however, it once again provided nonsensical results that could not be translated to values for the amplitude. A mobile phone's slow-motion tool was attempted for use and it worked surprisingly well as the pendulum's motion was not too fast for the camera to capture accurately and precisely. This method proved to be an effective method to measure the varying maximum amplitudes based on the surface area of the circular cardboard cutout attached to the pendulum bob.

<u>Procedure</u>

1. Set up the retort stand by attaching the G-clamp securely to a stable surface, such as a table or benchtop.
2. Attach the retort clamp to the retort stand arm, ensuring it is at a height suitable for the pendulum bob to swing freely.
3. Use the cross clamp to attach the thin thread to the hook of the pendulum bob.
4. Measure and record the length of the thin thread (0.3336 ± 0.002 m).
5. Prepare the 6 cardboard cutouts of circles with varying surface areas using a laser cutter. Make sure the dimensions and surface areas are accurately known and recorded.
6. Apply a small amount of super glue to the back of each cardboard cutout and carefully attach them to the pendulum bob, ensuring they are centered and securely glued.
7. Displace the pendulum bob by a known initial amplitude of 0.007 ± 0.001 meters from the equilibrium position and release it to allow it to oscillate freely.
8. Use the slow-motion tool on a phone to record the motion of the pendulum bob.
9. Analyze the recorded video to determine the maximum amplitude of the pendulum bob at the 8th oscillation. Record this value for each surface area.
10. Repeat the analysis for the 11th oscillation and record the corresponding maximum amplitude.
11. Measure and record the time duration between the 8th and 11th oscillation for each surface area.
12. Repeat the entire experiment for the remaining 5 cardboard cutouts with different surface areas.
13. Compile the recorded data, including the surface areas, maximum amplitudes at the 8th and 11th oscillation, and time duration between the 8th and 11th oscillation for each surface area.





Photographs taken by candidate

Environmental, ethical, and safety considerations

In order to maximize environmental sustainability, recycled cardboard was used for the circular cutouts of varying surface areas and the remaining cardboard that was not used was returned to be used in other applications. In terms of ethical considerations, the experiment was conducted after obtaining necessary permissions. Furthermore, there are various safety considerations to take note of upon conducting the experiment; firstly, it must be ensured that proper safety protocols are followed when operating the laser cutter to avoid injury or strain and when utilizing the super glue in the experimental procedure. In order to avoid the obstruction of property and harm of individuals nearby, pay careful attention to the trajectory of a pendulum when measuring the maximum amplitude.

Data Collection

Preliminary Processed Data Table

Table 1: Measurements for diameter of circular cardboard cutouts and their corresponding surface areas using accurate and precise laser cutting technology (**note**: first diameter recorded is of the initial pendulum bob using a micrometer screw gauge while the others were adapted from this value proportionally and inputted into the laser cutter with no uncertainty).

Diameter of circular cardboard cutout (m)	Surface Area of circular cardboard cutout (cm^2)
0.0252 ± 0.00000005	4.988
0.0302	7.163
0.0352	9.731
0.0402	12.692
0.0452	14.200
0.0502	19.792
0.0552	23.931

Sample calculation for uncertainty in micrometer screw gauge reading although it is practically negligible:

MSR (Main Scale Reading) + CSR (Circular Scale Reading) $\pm \frac{LC}{2}$

$= 2.50 + 0.02 \pm \frac{0.01}{2}$ cm $= 0.252 \pm 0.00000005$ m (converted to meters as it is the familiar SI unit)

Raw Data Tables

Table 2: Relationship between the surface area (cm^2) of a circular cardboard cut out attached to a pendulum and the maximum amplitude reached after 8 oscillations and 11 oscillations, as well as the time between the 8th and 11th oscillations.

Surface area (m^2)	**Trial number**	A_n (amplitude for nth cycle $\pm$ 0.001 m) (m) **where n = 8**	A_{n+m} (amplitude for nth + mth cycle $\pm$ 0.001 m) (m) **where m = 3**	**Time elapsed between** A_n **and** A_{n+m} $\pm$ 0.01 s (s)
4.988	1	0.096	0.085	3.98
	2	0.095	0.084	4.01
	3	0.095	0.084	4.00

7.163	1	0.083	0.075	3.02
	2	0.082	0.073	3.05
	3	0.082	0.074	3.09
9.731	1	0.088	0.071	3.58
	2	0.082	0.072	3.28
	3	0.086	0.071	3.34
12.692	1	0.076	0.067	3.47
	2	0.074	0.065	3.21
	3	0.077	0.064	3.33
14.200	1	0.066	0.059	3.22
	2	0.071	0.060	3.03
	3	0.067	0.060	3.11
19.792	1	0.065	0.058	3.11
	2	0.068	0.057	3.23
	3	0.067	0.057	3.12
23.931	1	0.059	0.052	3.03
	2	0.063	0.050	3.14
	3	0.059	0.051	3.17

The maximum amplitude reached at the 8th and 11th oscillations have been converted from centimeters (cm) which was measured to two significant figures to meters (m), which was measured to significant figures as well in order to conserve the precision of the original values. This conversion was done as meters are a more familiar unit to work with being an SI unit and can be integrated into the logarithmic decrement equation more easily as it functions with SI units only. Since the amplitudes were measured using a ruler, their uncertainty is equal to the smallest increment on the ruler (0.001 m) divided by two and multiplied by the points at which approximations were made (two). Therefore, $\frac{0.001}{2} \times 2 = \pm 0.001\, m$. Furthermore, the uncertainty in the time measured is to two decimal places based on the precision of the mobile phone timer.

Table 3: Qualitative observations of the relationship between the surface area (cm^2) of a circular cardboard cut out attached to a pendulum and the maximum amplitude reached after 8 oscillations and 11 oscillations, as well as the time between the 8th and 11th oscillations.

1	As greater surface areas were attached to the pendulum bob, the increased air resistance lead to more of an angular trajectory, and thus, the data points had to be repeated several times in order to obtain the most linear path trajectory
2	At times, the pendulum would be set off at varying velocities due to the friction between the hand displacing the pendulum bob and the bob itself; however, that was kept to a minimum by replicating the method of setting the pendulum bob off as much as possible throughout the trials.
3	As greater surface areas were attached to the pendulum bob, it was more difficult to record the amplitude as the cardboard cut out would cover the ruler used for reference; however, this was not much of an issue as it only required a change of angle of the mobile phone videoing the oscillations in order to extract a precise value through frame-by-frame analysis of the pendulum's trajectory.

Processed Data Tables

Table 4: Processed table showing the effect of varying surface areas (m^2) of a circular cardboard cut out attached to a pendulum and the averaged maximum amplitude values reached after 8 oscillations and 11 oscillations, as well as the averaged time values between the 8th and 11th oscillations on the damping coefficient.

Surface area (m^2)	**Average A_n** (amplitude for nth cycle $\pm$ 0.003 m) (m) **where n = 8**	**Average A_{n+m}** (amplitude for nth + mth cycle $\pm$ 0.003 m) (m) **where m = 3**	**Average time elapsed between A_n and A_{n+m}** $\pm$ 0.03 s (s)	**Coefficient of damping (b)**
4.998	0.095	0.084	3.99	-0.01028
7.163	0.082	0.074	3.05	-0.01121
9.731	0.085	0.071	3.40	-0.01304
12.692	0.075	0.065	3.34	-0.01428

14.200	0.068	0.059	3.12	-0.01516
19.792	0.067	0.057	3.19	-0.0168
23.931	0.061	0.051	3.11	-0.01919

Sample calculation for average A_n (amplitude for nth cycle $\pm$ 0.001 m) value for when n = 8 and repeated with similar methods for average :

$$average = \frac{sum\ of\ trials\ values}{number\ of\ trials}$$

$$= \frac{T_1+T_2+T_3}{3} = \frac{0.096+0.095+0.095}{3} = 0.095\ m$$

(to two significant figures and three decimal places)

Sample calculation for coefficient of damping force (b) using averaged values for A_n, A_{n+m}, and time elapsed between A_n and A_{n+m}:

$$b = \frac{ln(A_{n+m}/A_n)}{mT}$$

$$b = \frac{ln(\frac{0.084}{0.095})}{3\times3.99} = -\ (0.01028)$$

(to four significant figures and five decimal places)

Table 5 (Part 1): Uncertainties of the processed data table showing the effect of varying surface areas (cm^2) of a circular cardboard cut out attached to a pendulum and the averaged maximum amplitude values reached after 8 oscillations and 11 oscillations, as well as the averaged time values between the 8th and 11th oscillations on the damping coefficient.

Surface Area (m^2)	**Trial**	A_n **(m)**	A_{n+m} **(m)**	**Time (s)**	**Absolute Uncertainty** A_n **(m)**	**Fractional Uncertainty** A_n **(m)**	**Absolute Uncertainty** A_{n+m} **(m)**	**Fractional Uncertainty** A_{n+m} **(m)**	**Absolute Uncertainty Time (s)**	**Fractional Uncertainty Time**
4.988	1	0.096	0.085	3.98	0.005	0.052	0.005	0.059	0.01	0.003
	2	0.095	0.084	4.01	0.005	0.053	0.005	0.059	0.01	0.002
	3	0.095	0.084	4.00	0.005	0.053	0.005	0.059	0.01	0.003

7.163	1	0.083	0.075	3.02	0.006	0.072	0.006	0.080	0.04	0.013
	2	0.082	0.073	3.05	0.006	0.073	0.006	0.083	0.04	0.013
	3	0.082	0.074	3.09	0.006	0.073	0.006	0.083	0.04	0.013
9.731	1	0.088	0.071	3.58	0.008	0.089	0.008	0.112	0.20	0.006
	2	0.082	0.072	3.28	0.008	0.098	0.008	0.111	0.20	0.006
	3	0.086	0.071	3.34	0.008	0.094	0.008	0.112	0.20	0.006
12.692	1	0.076	0.067	3.47	0.009	0.119	0.009	0.134	0.01	0.003
	2	0.074	0.065	3.21	0.009	0.122	0.009	0.138	0.01	0.003
	3	0.077	0.064	3.33	0.009	0.117	0.009	0.141	0.01	0.003
14.200	1	0.066	0.059	3.22	0.004	0.061	0.004	0.068	0.01	0.003
	2	0.071	0.060	3.03	0.004	0.056	0.004	0.066	0.01	0.003
	3	0.067	0.060	3.11	0.004	0.060	0.004	0.066	0.01	0.003
19.792	1	0.065	0.058	3.11	0.003	0.046	0.003	0.051	0.1	0.030
	2	0.068	0.057	3.23	0.003	0.044	0.003	0.047	0.1	0.030
	3	0.067	0.057	3.12	0.003	0.045	0.003	0.048	0.1	0.030
23.931	1	0.059	0.052	3.03	0.003	0.051	0.003	0.058	0.01	0.003
	2	0.063	0.050	3.14	0.003	0.048	0.003	0.060	0.01	0.003

	3	0.059	0.051	3.17	0.003	0.051	0.003	0.058	0.01	0.003

Sample calculations for the absolute uncertainties in the table above:

$$absolute\ uncertainty\ = \frac{max-min}{2}$$
$$= \frac{4.01-3.98}{2} = 0.01\ m$$

to one significant figure and two decimal places

Sample calculations for the fractional uncertainties in the table above:

$$fractional\ uncertainty\ = \frac{absolute\ uncertainty}{magnitude\ of\ quantity}$$
$$= \frac{0.005}{0.096} = 0.052\ m$$

to two significant figures and three decimal places

Table 5 (Part 2):

Uncertainties of the processed data table for the damping coefficient through the use of the averaged A_n (m), A_{n+m} (m) , and time (s) data and their corresponding fractional uncertainty data.

Surface area (cm^2)	**Absolute uncertainty in average** A_n (amplitude for nth cycle $\pm$ 0.003 m) (m) **where n = 8**	**Absolute uncertainty in average** A_{n+m} (amplitude for nth + mth cycle $\pm$ 0.003 m) (m) **where m = 3**	**Absolute uncertainty in average time elapsed between** A_n **and** A_{n+m} $\pm$ 0.03 s (s)	**Coefficient of damping force (b)**	**Absolute uncertainty in coefficient of damping force (b)**	**Fractional uncertainty in coefficient of damping force (b)**
4.998	0.003	0.003	0.03	-0.01028	0.036	35.02
7.163	0.003	0.003	0.03	-0.01121	0.036	32.11
9.731	0.003	0.003	0.03	-0.01304	0.036	27.61
12.692	0.003	0.003	0.03	-0.01428	0.036	2.52
14.200	0.003	0.003	0.03	-0.01516	0.036	2.37
19.792	0.003	0.003	0.03	-0.01680	0.036	2.14

23.931	0.003	0.003	0.03	-0.01919	0.036	1.88

Sample calculation for absolute uncertainty in the coefficient of damping force (b):

$absolute\ uncertainty\ =\ \Delta A_n\ +\ \Delta A_{n+m} +$
$\Delta\ time\ elapsed\ between\ A_n\ and\ A_{n+m}$
$=\ 0.003\ +\ 0.003\ +\ 0.3\ =\ 0.036$

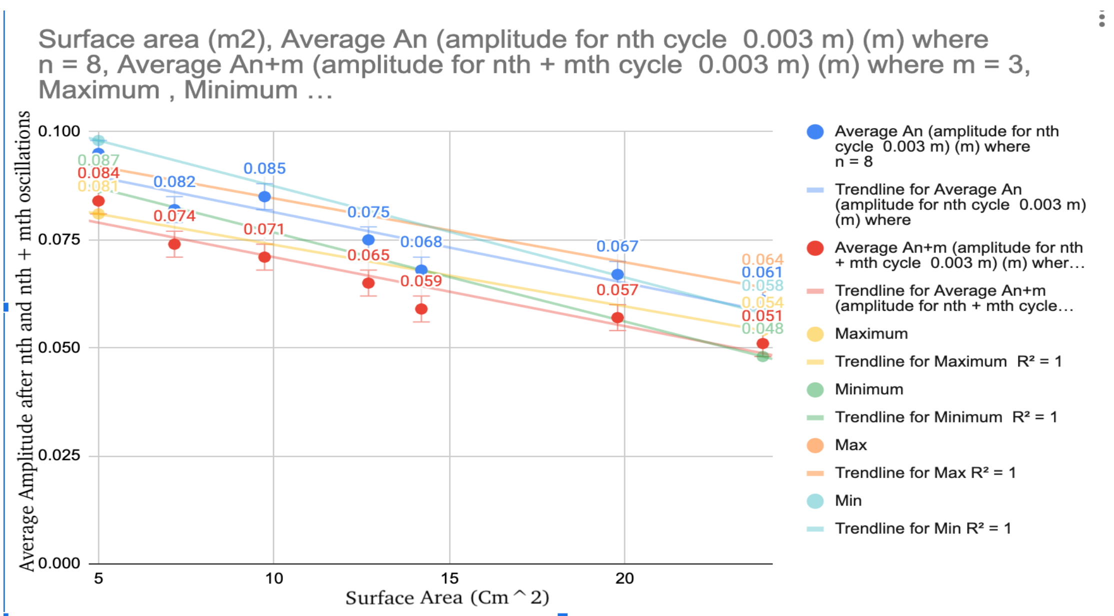
Sample calculation for finding fractional uncertainty in the coefficient of damping force (b):

$fractional\ uncertainty\ =\ \frac{absolute\ uncertainty}{magnitude\ of\ quantity}$

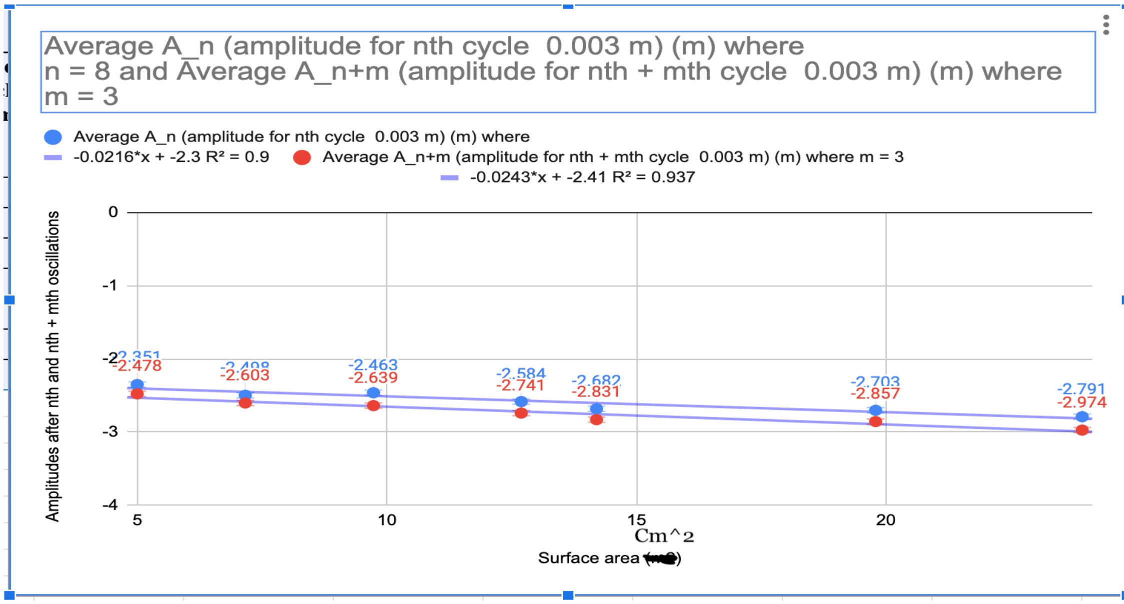
$\frac{\Delta b}{b}\ =\ \frac{0.036}{0.01028}\ =\ 32.02$

Processed data in graphical form

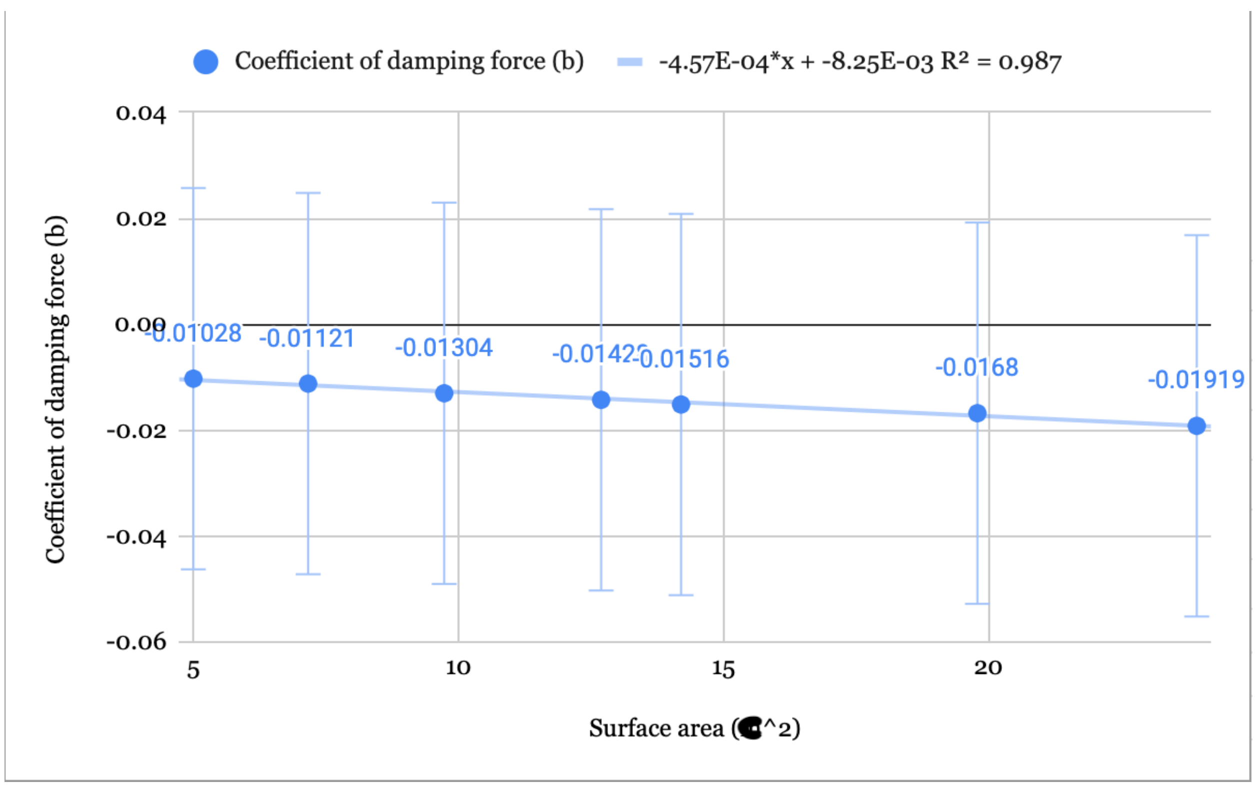
Graph 1: Relationship between surface area of a circular cardboard cut out attached to a pendulum bob and average amplitude after eight oscillations or after it reaches A_n (blue) and after three more oscillations or after it reaches $A_{n+m}(red)$.



Graph 2: Relationship between surface area of a circular cardboard cut out attached to a pendulum bob and the natural log of the average amplitude after eight oscillations or after it reaches A_n (blue) and after three more oscillations or after it reaches $A_{n+m}(red)$



Graph 3: Relationship between surface area of a circular cardboard cut out attached to a pendulum bob and value for damping coefficient.



Conclusion

The purpose of this investigation was to address the research question: *To what extent does the surface area attached to a pendulum bob have an affect on the damping coefficient?*

To preface, the max-min lines were left out from **graph 2** and **graph 3** for different reasons. In **graph 2**, the max-min slope lines are left out as they do not contribute to the overall analysis of the relationship. The reason for this is that the uncertainty is extremely small (0.003) compared to the scale of the graph and thus the uncertainty works out to be similarly negligible. In regards to **graph 3**, the max-min slope lines would not maintain the scientific validity of the report if they were to be included as the uncertainty suggests that the damping coefficient may have a positive value which cannot be as that would imply that the amplitudes could increase as the surface area is increased which is undoubtedly untrue; therefore, they were omitted for this reason.

The experimental results indicate that there exists a negative natural logarithmic relationship between surface area (cm^2) and the respective amplitudes A_n and A_{n+m} as the relationship was first attempted to be modeled in a linear fashion; however, its correlation coefficient, r, was not maximized with the linear trend line. This is evident in **graph 1** where it is clear to see that the line of best fit passes through few of the given data points. Therefore, through a combination of trial and error and rational thinking, it was decided upon to model this relationship between the surface area (cm^2) and the <u>natural logarithm</u> of the respective amplitudes A_n and A_{n+m} in an attempt to linearize the relationship. This proved to be fruitful as a close to proportional relationship was thus uncovered between these two variables with negligible uncertainties as discussed before. It should be noted that that although **graph 1** did not have a perfect line of best fit, it's overall negative correlation aligns with the theoretical background discussed that as a result of increasing the surface area attached to the pendulum bob, the system would experience greater damping forces, thereby decreasing the maximum amplitude that a simple pendulum setup could reach compared to an ideally undamped simple pendulum model. **Graph 2** is similarly effective theoretically as it includes a strong negative correlation between surface area (cm^2) and the natural logarithm of the amplitudes A_n and A_{n+m} reached at the 8th and the 11th oscillation respectively.

Both strong negative correlations explored in **graph 1** and **graph 2** are further supported and explored in greater detail upon considering **graph 3**. **Graph 3** illustrates the relationship between surface area (cm^2) and the damping coefficient (b); the main exploration of this investigation. Similarly to the hypothesis made about their relationship, the graph demonstrates that an increase in surface area leads to a more negative value for the damping coefficient. This translates to an increased presence of damping forces in the pendulum system as a result of the increased frequency of collision of air molecules, hence resulting in greater air resistance as hypothesized. Therefore, in that sense, the initial hypothesis has been proven and the relationship between each factor introduced in the logarithmic decrement equation has been explored.

Evaluation

There were several procedural strengths that deserve to be addressed prior to moving onto the limitations and areas of growth of the experiment.

1. All the controlled variables were monitored extremely closely such that the independent variable, surface area (cm^2), would be able to be isolated from other possible extraneous or confounding variables. Thus, the relationships between the surface area of the circular cardboard cut out attached to the pendulum bob and the dependent variables of interest (i.e. coefficient of damping force and measured amplitudes after 8 and 11 oscillations) can be attributed to the manipulation of the independent variable rather than other confounding factors. The experimental procedure was kept consistent through maintaining several controlled variables such as the length of the string, the mass of the pendulum bob, and the initial displacement. Furthermore, all the experimental data was extracted in the same location in order to minimize fluctuations in the immediate surroundings of the pendulum system. This area of strength has been reflected in the graphical data illustrated above due to the fact that there is an absence of anomalous data points. Moreover, the minimization of such systematic and random errors increases the accuracy and overall reliability of the results.
2. The slow-motion tool used in the device throughout the experiment provided measurements for the pendulum bob's maximum displacements, A_n and A_{n+m} respectively, from a set point of equilibrium at the 8th and 11th oscillations, to a high degree of accuracy of precision as it utilizes frame-by-frame software to analyze the trajectory of the pendulum bob. Therefore, the extracted data values for the respective amplitudes were not constricted with instrumental uncertainties, but rather, the uncertainty associated with the ambiguity of human readings which cannot be exactly quantified. Moreover, the absolute uncertainties in the average amplitudes were extremely low, ranging from $\pm$ 0.003 while the uncertainty in the average time between the nth and mth oscillation was a mere $\pm$ 0.03. As a result, this provided relatively consistent data points with an added benefit of little to no random errors; further reinforcing the precision of the results.
3. The independent variable was manipulated for seven different readings for the surface area of the circular cardboard cut out attached to the pendulum bob, 4.998, 7.163, 9.731, 12.692, 14.200, 19.792, 23.931 cm^2, and the dependent variable was measured throughout seven different and corresponding readings. Therefore, there was a wide range of data collected in order to identify a clear pattern between the two variables that cannot be merely referred to as situational proportionality. The reason why it is kept in centimeters instead of meters is that the values would have become extremely small and that would sacrifice the fluidity of the lab report due to the fact that it will become hard to read and follow with the exceptionally miniscule numbers so an exception is made for not including the SI unit in this case.

Nevertheless, there are still several uncertainties associated with the data arising from the various limitations of the experimental setup and procedure that must be addressed as well.

1. Various times upon collecting the data of the respective maximum amplitudes A_n and A_{n+m}, the pendulum often oscillated in a circular trajectory that could not be accounted

for within the scopes of the experiment. This is a significant weakness since the dissipative forces being measured in the form of the coefficient of damping force may have been affected due to the fact that the air molecules were were not resisting the pendulum's motion completely, especially when measuring the maximum amplitudes for the pendulum bobs with large surface circular cardboard cut out surfaces attached to them.

2. The moment of displacement of the pendulum bob was also subject, at times, to increased force as a result of the instant frictional force between the hand displacing the pendulum bob and the pendulum bob, which may have affected recorded data as it increased the velocity of release of the pendulum bob. Therefore, this factor may have affected the results slightly as it was minimized throughout the experiment by ensuring that I did not release the pendulum bob rashly but rather gently in order to minimize the aforementioned frictional force.

In order to gain a better understanding of the factors that affect the coefficient of damping forces experienced by a pendulum system, the effects that different viscous liquids have on the trajectory of a pendulum, and therefore, damping coefficient may also be investigated. Similarly, the relationship between the temperature of surrounding air molecules and the damping coefficient may also be investigated as another alternative that holds real-life applications. Either of these alternatives are important to explore further as, in the case of experimenting with different viscous liquids, it is important to engineers to select appropriate liquids in order to optimize the performance on structures or machines depending on their durability or expected output. Furthermore, the relationship between temperature and the damping coefficient is another alternative and practical experiment that is important due to the fact that it is utilized in a structural sense in terms of assisting engineers in designing structures that maintain a balance between their dissipative forces in order to mitigate vibrations as a result of varying degrees of temperature (CMACN).

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