I do not understand this key step in proving Cauchy-Schwartz Inequality. Most appreciated if someone could help!
(From book Quantum Mechanics DeMysrified Pg. 133)

EXAMPLE 5-8

The *Cauchy-Schwartz inequality* states that

$$|\langle\phi|\psi\rangle|^2 \leq \langle\psi|\psi\rangle\langle\phi|\phi\rangle$$

Prove this result.

SOLUTION

The proof also relies on the fact that $\langle f|f\rangle \geq 0$ for any vector. We car minimize this expression if we let

I do not understand this step

$$|f\rangle = |\phi\rangle - \frac{\langle\psi|\phi\rangle}{\langle\phi|\phi\rangle}|\psi\rangle$$

and so we have

$$\langle f|f\rangle = \langle\phi|\phi\rangle - \frac{\langle\psi|\phi\rangle}{\langle\psi|\psi\rangle}\langle\phi|\psi\rangle - \frac{\langle\phi|\psi\rangle}{\langle\psi|\psi\rangle}\langle\psi|\phi\rangle + \frac{\langle\phi|\psi\rangle}{\langle\psi|\psi\rangle}\frac{\langle\psi|\phi\rangle}{\langle\psi|\psi\rangle}\langle\psi|\psi\rangle$$

Now we use the fact that

$$\langle\phi|\psi\rangle\langle\psi|\phi\rangle = |\langle\phi|\psi\rangle|^2$$

to rewrite this as

$$\langle f|f\rangle = \langle\phi|\phi\rangle - 2\frac{|\langle\phi|\psi\rangle|^2}{\langle\psi|\psi\rangle} + \frac{|\langle\phi|\psi\rangle|^2}{\langle\psi|\psi\rangle} = \langle\phi|\phi\rangle - \frac{|\langle\phi|\psi\rangle|^2}{\langle\psi|\psi\rangle}$$