

Relevant Equations:

$$E = \frac{kq}{r^2}$$

$$\Delta V = - \vec{E}r$$

$$\Phi = \int \vec{E} \vec{A} = \frac{q_{enc}}{\epsilon_0}$$

Process:

First, I used Gauss' Law to try to determine the electric field with respect to the line charge density.

$$\Phi = \int \vec{E} \vec{A} = \frac{q_{enc}}{\epsilon_0}$$

For constant, E and A:

$$\Phi = EA = \frac{q_{enc}}{\epsilon_0}$$

Using a cylinder with radius, r and length L, for the Gaussian surface:

$$E * 2\pi rL = \frac{q_{enc}}{\epsilon_0}$$

To determine the enclosed charge:

$$q_{enc} = \lambda L$$

$$E * 2\pi rL = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r\epsilon_0}$$

Now to determine the electric field with respect to voltage:

$$\Delta V = - \vec{E}r$$

$$\vec{E} = - \frac{\Delta V}{r}$$

Now bringing them together:

$$-\frac{\Delta V}{r} = \frac{\lambda}{2\pi r\epsilon_0}$$

$$\lambda = - \frac{\Delta V * 2\pi r\epsilon_0}{r}$$

$$\lambda = - \Delta V * 2\pi\epsilon_0$$

So let's plug in the numbers now,

$$\lambda = - (3.9 \text{ kV} * \frac{1000 \text{ V}}{1 \text{ kV}}) * 2\pi(8.854 * 10^{-12} \frac{\text{C}^2}{\text{N} * \text{m}^2})$$

Let's make the units a little nicer to work with to help me conceptualize a bit,

$$\begin{aligned}\lambda &= - 3900\text{V} * 2\pi(8.854 * 10^{-12} \frac{\text{C}^2}{\text{N} * \text{m}^2}) \\ \lambda &= - 3900 \frac{\text{N} * \text{m}}{\text{C}} * 2\pi(8.854 * 10^{-12} \frac{\text{C}^2}{\text{N} * \text{m}^2}) \\ \lambda &= 2.17\text{E} - 7 \frac{\text{C}}{\text{m}} * \frac{1\text{E}9 \text{ nC}}{1 \text{ C}} = - 216.96 \frac{\text{nC}}{\text{m}}\end{aligned}$$

But that's not right according to the homework software I'm using, so I'm not exactly sure where I made my mistake. I know that the diameter and distance don't show up at all in the final equation which seems strange but I don't know how to incorporate them into the final equation because the final units match up with what we would expect for line charge density.