

The only free variables are δp , δu_r , δu_θ , δu_ϕ , δr_{md} , and $\delta \epsilon$ defined in terms of δp , which is in turn a fn of r , θ that NDSolve is supposed to find. The variables α , β , ν , ω , ϵ and p , are found from using Interpolation. i.e. $\alpha = \text{Interpolation}[\alpha]$. Where α is my data. The $d\alpha = \text{Function}[r, \theta, \text{Evaluate}[D[\alpha, r]]]$. Similarly for the other $d...r$ or $d... \theta$ terms. $m = 2$. u is some function of r and θ . Capital ω is just a number.

The only variable that I'm not sure about is σ , and I thought this was throwing errors since I hadn't defined it in any way. So I just tried $\sigma = 10$, and it still gave me the same error. I'm also not sure if my BC are right. Could that be causing the issue? I was getting BC errors before but after playing around with the BC those errors went away and I was left with this one.

Once again with hopefully with more clarity, is the command that I've giving mathematica:

```
sol = NDSolve[

{

D[ $\delta u_r[r, \theta], r$ ] + D[ $\delta u_\theta[r, \theta], \theta$ ] == (u/rmd[r,  $\theta$ ] * ( $\sigma - m\Omega$ ) *  $\delta r_{md}[r, \theta]$ ) -
(2/r + 1/rmd[r,  $\theta$ ] *  $\delta r_{mdr}[r, \theta]$  + 2dalphar[r,  $\theta$ ] + dbetar[r,  $\theta$ ] +  $\delta u_r[r, \theta]$ ) *  $\delta u_r[r, \theta]$  -
(Cot $\theta$  + 1/rmd[r,  $\theta$ ] *  $\delta r_{md\theta}[r, \theta]$  + 2dalpha $\theta$ [r,  $\theta$ ] + dbeta $\theta$ [r,  $\theta$ ] +  $\theta[r, \theta]$ ) *  $\delta u_\theta[r, \theta]$ 
+ ( $\sigma$  * F[r,  $\theta$ ] - m) *  $\delta u_\phi[r, \theta]$ ,

D[ $\delta p[r, \theta], r$ ] == (1 / ( $\epsilon + p$ ) * D[p[r,  $\theta], r]$ ) * ( $\delta \epsilon[r, \theta]$  +  $\delta p[r, \theta]$ ) - (( $\epsilon + p$ ) * u) / Exp[-2
 $\alpha$ ] * ( $\sigma - m\Omega$ ) *  $\delta u_r[r, \theta]$  + (( $\epsilon + p$ ) * u) / Exp[-2  $\alpha$ ] * (Exp[2  $\beta - 2 \alpha$ ] *  $r^2$  * (Sin $\theta$ )2 * ( $\Omega - \omega[r, \theta]$ ) * D[Log[F[r,  $\theta], r]$ ] *  $\delta u_\phi[r, \theta]$ ),

D[ $\delta p[r, \theta], \theta$ ] == (1 / ( $\epsilon + p$ ) * D[p[r,  $\theta], \theta]$ ) * ( $\delta \epsilon[r, \theta]$  +  $\delta p[r, \theta]$ ) - (( $\epsilon + p$ ) *  $r^2$  *
u) / Exp[-2 $\alpha$ ] * ( $\sigma - m\Omega$ ) *  $\delta u_\theta[r, \theta]$  + (( $\epsilon + p$ ) *  $r^2$  * u) / Exp[-2 $\alpha$ ] * (Exp[2  $\beta - 2 \alpha$ ] *
 $r^2$  * (Sin $\theta$ )2 * ( $\Omega - \omega[r, \theta]$ ) * D[Log[F[r,  $\theta], r]$ ] *  $\delta u_\phi[r, \theta]$ ),

(*Boundary Conditions*)
 $\delta \theta[1, \theta] == 0$ ,  $\delta u_r[1, \theta] == \delta p[1, \theta] == \delta \theta[r, 1] == \delta u_r[r, 1] == \delta p[r, 1] == 0$ , -I* $\gamma$ 1* $\delta p[128,$ 
 $\theta + \delta u_r[128, \theta]$  * Evaluate[D[ $\delta p[128, \theta], r]$ ] +  $\delta u_\theta[128, \theta]$  * Evaluate[D[ $\delta p[128, \theta], \theta]$ ] == 0,

(*what I'm solving for, and the bounds*)
{ $\delta p, \delta u_r, \delta u_\theta$ }, {r, 1, 128}, { $\theta$ , 1, 64}}
```