

Calculating flow out of an open pipe using Bernoulli, Darcy-Weisbach, Blasius

Bernoulli's Equation, solve for v_2 (ignoring height change component)

$$P_1 + \frac{1}{2} \cdot \rho_{fluid} \cdot v_1^2 = (P_2 - \Delta P) + \frac{1}{2} \cdot \rho_{fluid} \cdot v_2^2 \xrightarrow[\text{assume, } \rho_{fluid} > 0]{\text{solve, } v_2} \left[\frac{\sqrt{\rho_{fluid} \cdot v_1^2 + 2 \cdot \Delta P + 2 \cdot P_1 - 2 \cdot P_2}}{\sqrt{\rho_{fluid}}} \right]$$

Darcy-Weisbach equation, solve for ΔP :

$$\frac{\Delta P}{L_{pipe}} = f_D \cdot \frac{\rho_{fluid}}{2} \cdot \frac{v_{avg}^2}{D_{pipe}} \xrightarrow{\text{solve, } \Delta P} \frac{f_D \cdot v_{avg}^2 \cdot L_{pipe} \cdot \rho_{fluid}}{2 \cdot D_{pipe}}$$

Blasius equation for f , curve fit for smooth pipe $2100 < Re < 100000$

$$D_{pipe} := 15 \text{ mm}$$

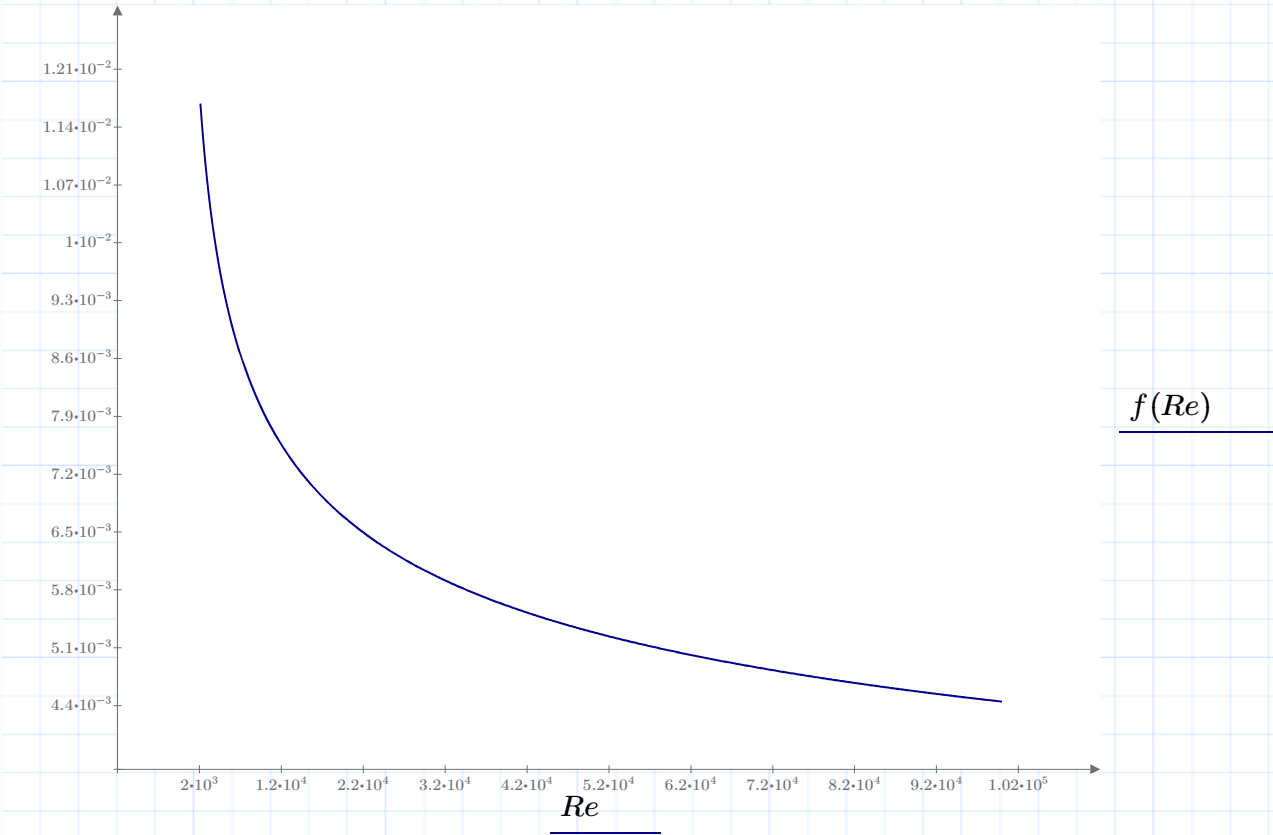
$$\nu_{water} := 1.08 \cdot 10^{-5} \frac{ft^2}{s} = (1.003 \cdot 10^{-6}) \frac{m^2}{s}$$

$$f(Re) := \frac{0.0791}{Re^{0.25}}$$

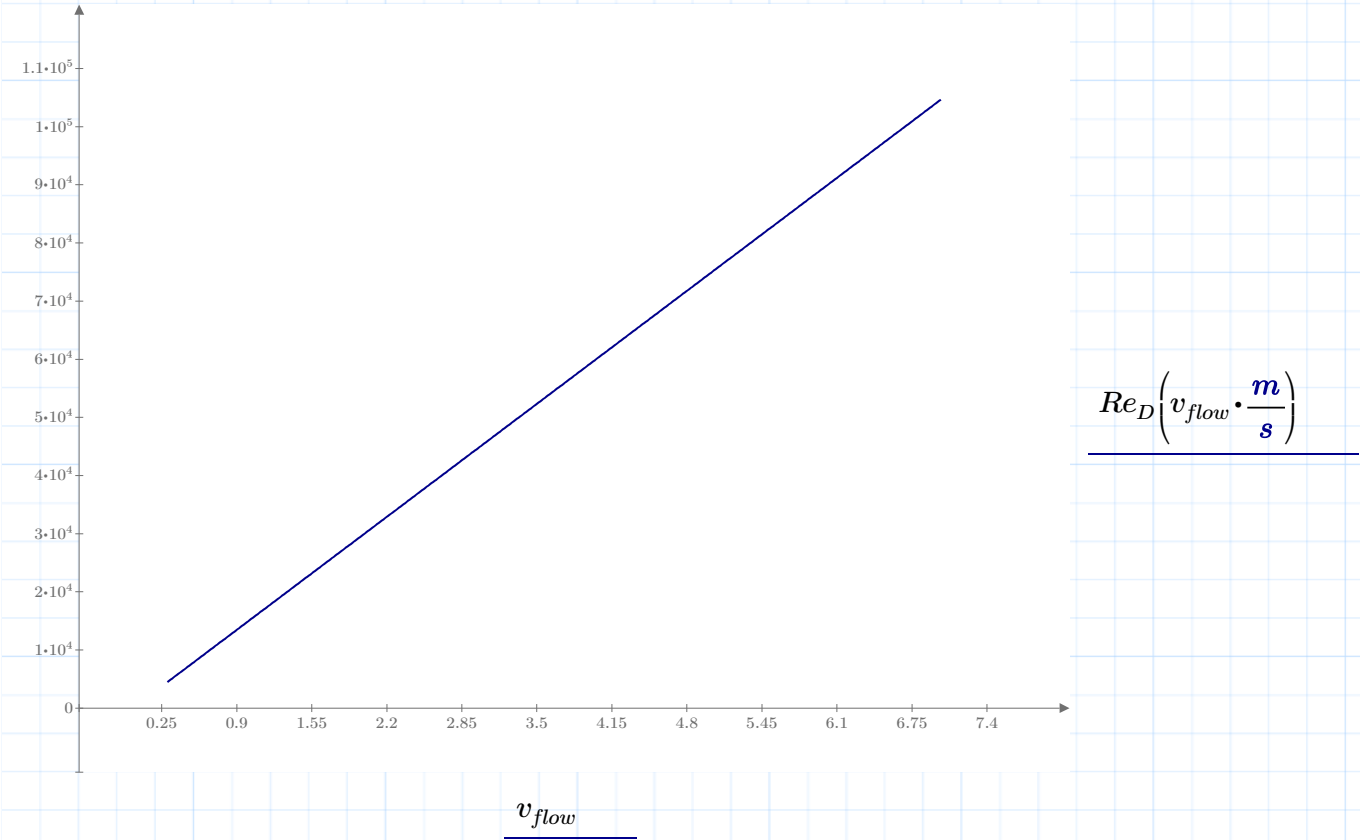
$$Re_D(v_{avg}) := \frac{v_{avg} \cdot D_{pipe}}{\nu_{water}}$$

$$f_D(v_{avg}) := \frac{0.0791}{(Re_D(v_{avg}))^{0.25}}$$

Friction Coefficient vs Reynold's Number, Blasius Equation



Flow Velocity vs. Reynold's Number, Using Pipe Dimensions



Pipe flow parameters:

$$L_{pipe} := 20 \text{ m}$$

$$\nu_{water} := 1.08 \cdot 10^{-5} \frac{ft^2}{s} = (1.003 \cdot 10^{-6}) \frac{m^2}{s}$$

$$\rho_{water} := 1000 \frac{kg}{m^3} \quad \text{Water Density}$$

$$v_1 := 0 \frac{m}{s} \quad \text{Initial Velocity}$$

$$P_1 := 5 \text{ bar} \quad \text{Pressure at start of pipe}$$

$$P_2 := 1 \text{ atm} \quad \text{Atmospheric Pressure}$$

$$D_{pipe} := 15 \text{ mm} \quad \text{Pipe Diameter}$$

$$A_{pipe} := \frac{\pi \cdot D_{pipe}^2}{4} = 176.715 \text{ mm}^2 \quad \text{Calculate Pipe Exit Area}$$

Iterative Solve Block to find flow out of pipe (balance between flow rate and pressure loss due to flow in pipe):

Guess Values

$$v_2 := 1 \frac{m}{s}$$

$$V_{dot} := 1 \frac{m^3}{hr}$$

$$P_{loss} := 1 \text{ bar}$$

$$Re_D := 8.5 \cdot 10^4$$

$$f_D := 0.018$$

Guess values for iterative solver, numeric solutions shown below.

Constraints

$$(P_1 - P_{loss}) = P_2 + \frac{1}{2} \cdot \rho_{water} \cdot v_2^2$$

Bernoulli eqn., calculating flow from pressure difference. Model assumes P_1 pressure is reduced by P_{loss} due to flow friction

$$\frac{P_{loss}}{L_{pipe}} = f_D \cdot \frac{\rho_{water}}{2} \cdot \frac{v_2^2}{D_{pipe}}$$

Darcy-Weisbach Eqn.

$$f_D = \frac{0.0791}{Re_D^{0.25}}$$

Blasius Eqn. for estimating f_D from Re_D .

$$Re_D = \frac{v_2 \cdot D_{pipe}}{\nu_{water}}$$

Re_D calculation for pipe flow

$$V_{dot} = A_{pipe} \cdot v_2$$

Volume flow calculation for pipe flow

Solver

$$\text{Ans} := \text{find} (v_2, V_{dot}, P_{loss}, Re_D, f_D) = \begin{bmatrix} 11.343 \frac{m}{s} \\ 0.002 \frac{m^3}{s} \\ (3.343 \cdot 10^5) \text{ Pa} \\ 1.696 \cdot 10^5 \\ 0.004 \end{bmatrix}$$

Iterative Solver Solution:

$$v_{2_sol} := \text{Ans}_0 = 11.343 \frac{m}{s}$$

Calculated flow speed

$$V_{dot_sol} := \text{Ans}_1 = 7.216 \frac{m^3}{hr}$$

*Calculated volume flow rate
(m³/hr and gpm shown)*

$$V_{dot_sol} = 31.771 \text{ gpm}$$

$$P_{loss_sol} := \text{Ans}_2 = 3.343 \text{ bar}$$

Calculated pressure loss due to flow rate in pipe

$$Re_{D_sol} := \text{Ans}_3 = 1.696 \cdot 10^5$$

Calculated flow Reynold's number

$$f_{D_sol} := \text{Ans}_4 = 0.004$$

Calculated Darcy friction coefficient from Blasius Eqn.