Calculating flow out of an open pipe using Bernoulli, Darcy-Weisbach, Blasius

Bernoulli's Equation, solve for v_2 (ignoring height change component)

$$P_1+\frac{1}{2}\cdot\rho_{fluid}\cdot{v_1}^2 = \left(P_2-\Delta P\right)+\frac{1}{2}\cdot\rho_{fluid}\cdot{v_2}^2 \xrightarrow[]{\substack{solve,v_2\\ assume,\rho_{fluid}>0}} \begin{bmatrix} \dfrac{\sqrt{\rho_{fluid}\cdot{v_1}^2+2\cdot\Delta P+2\cdot P_1-2\cdot P_2}}{\sqrt{\rho_{fluid}}} \\ -\dfrac{\sqrt{\rho_{fluid}\cdot{v_1}^2+2\cdot\Delta P+2\cdot P_1-2\cdot P_2}}{\sqrt{\rho_{fluid}}} \end{bmatrix}$$

Darcy–Weisbach equation, solve for ΔP:

$$\frac{\Delta P}{L_{pipe}} = f_D\cdot\frac{\rho_{fluid}}{2}\cdot\frac{{v_{avg}}^2}{D_{pipe}} \xrightarrow[]{solve,\Delta P} \frac{f_D\cdot{v_{avg}}^2\cdot L_{pipe}\cdot\rho_{fluid}}{2\cdot D_{pipe}}$$

Blasius equation for Fanning friction factor (f), curve fit for smooth pipe 2100 < Re < 100000

$$D_{pipe}:=15\ mm$$

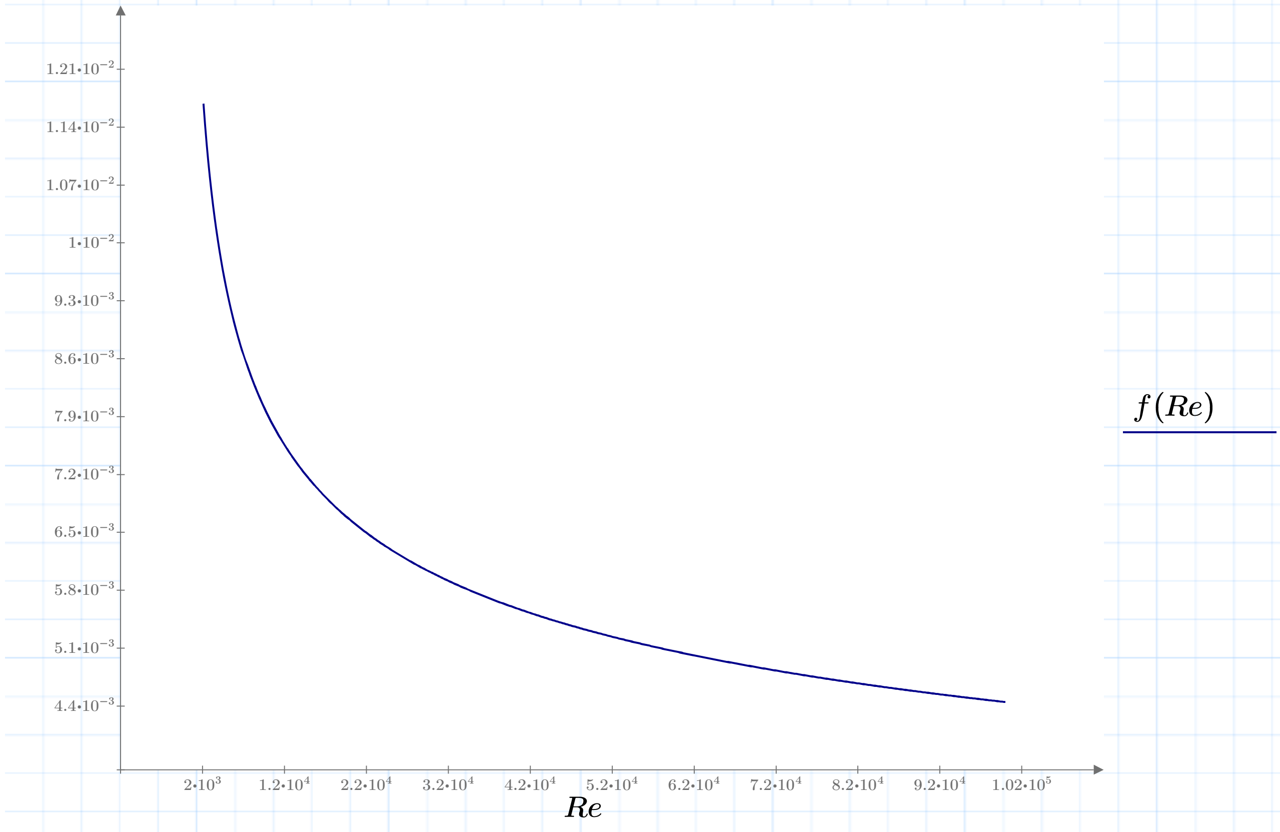
$$\nu_{water}:=1.08\cdot10^{-5}\ \frac{ft^2}{s}=\left(1.003\cdot10^{-6}\right)\ \frac{m^2}{s}$$

$$f(Re):=\frac{0.0791}{Re^{0.25}}$$

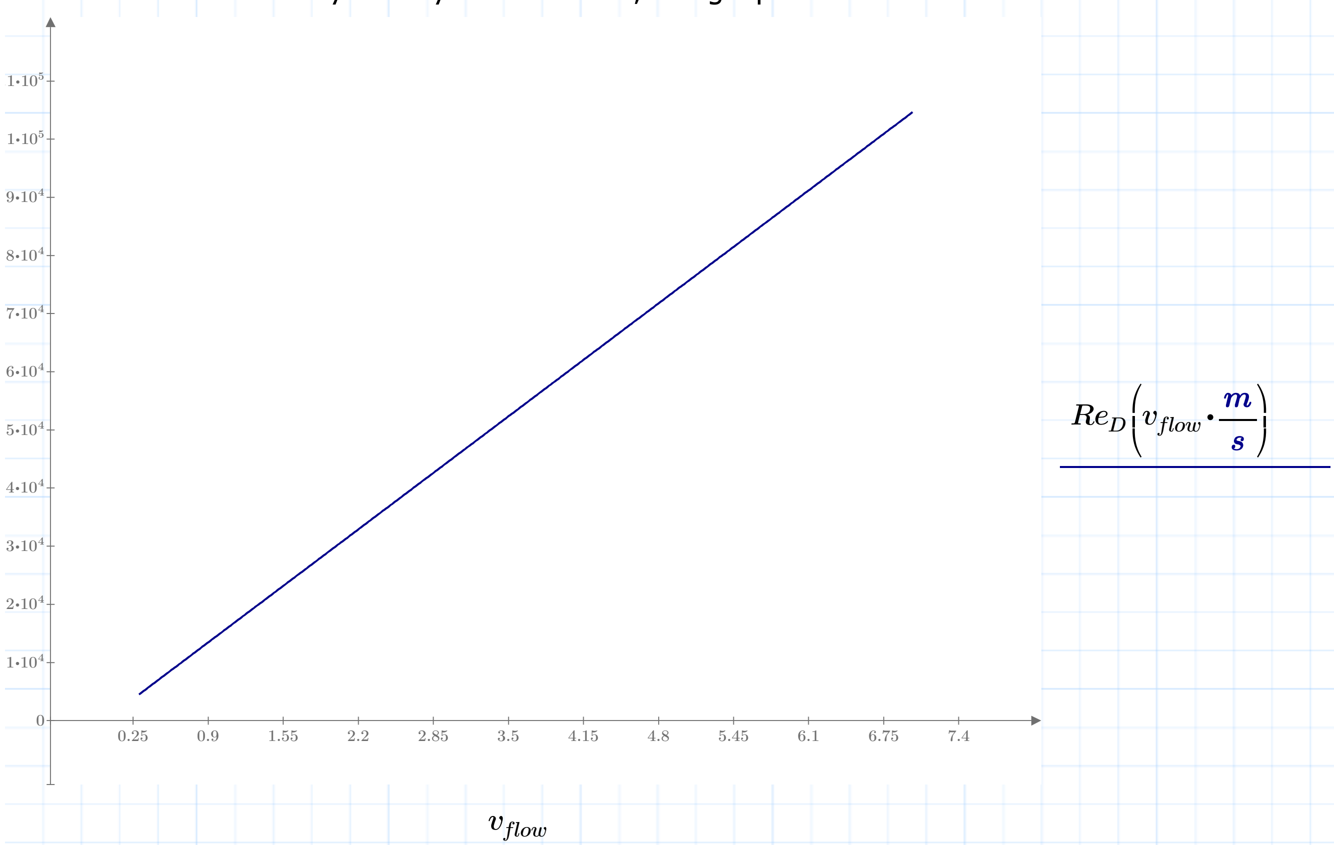
Blasius equation calculates the Fanning friction factor f. The Darcy friction factor f_D is 4x the Fanning friction factor.

$$Re_D\left(v_{avg}\right):=\frac{v_{avg}\cdot D_{pipe}}{\nu_{water}}$$

Friction Coefficient vs Reynold's Number, Blasius Equation



Flow Velocity vs. Reynold's Number, Using Pipe Dimensions



Pipe flow parameters:

$L_{pipe} := 20\ m$

$\nu_{water} := 1.08 \cdot 10^{-5}\ \frac{ft^2}{s} = \left(1.003 \cdot 10^{-6}\right)\ \frac{m^2}{s}$

$\rho_{water} := 1000\ \frac{kg}{m^3}$ *Water Density*

$v_1 := 0\ \frac{m}{s}$ *Initial Velocity*

$P_1 := 5\ bar$ *Pressure at start of pipe*

$P_2 := 1\ atm$ *Atmospheric Pressure*

$D_{pipe} = 15\ mm$ *Pipe Diameter*

$A_{pipe} := \frac{\pi \cdot D_{pipe}^{\ 2}}{4} = 176.715\ mm^2$ *Calculate Pipe Exit Area*

Iterative Solve Block to find flow out of pipe (balance between flow rate and pressure loss due to flow in pipe):

Guess Values

$$v_2 := 1\ \frac{m}{s}$$

$$V_{dot} := 1\ \frac{m^3}{hr}$$

$$P_{loss} := 1\ bar$$

$$Re_D := 8.5 \cdot 10^4$$

$$f_D := 0.018$$

Guess values for iterative solver, numeric solutions shown below.

Constraints

$$\left(P_1 - P_{loss}\right) = P_2 + \frac{1}{2} \cdot \rho_{water} \cdot {v_2}^2$$

Bernoulli eqn., calculating flow from pressure difference. Model assumes P_1 pressure is reduced by P_{loss} due to flow friction

$$\frac{P_{loss}}{L_{pipe}} = f_D \cdot \frac{\rho_{water}}{2} \cdot \frac{{v_2}^2}{D_{pipe}}$$

Darcy-Weisbach Eqn.

$$f_D = 4 \cdot \frac{0.0791}{{Re_D}^{0.25}}$$

Blasius Eqn. for estimating f from Re_D. Converted to Darcy factor using factor of 4.

$$Re_D = \frac{v_2 \cdot D_{pipe}}{\nu_{water}}$$

Re_D calculation for pipe flow

$$V_{dot} = A_{pipe} \cdot v_2$$

Volume flow calculation for pipe flow

Solver

$$\mathbf{Ans} := \mathbf{find}\left(v_2, V_{dot}, P_{loss}, Re_D, f_D\right) = \begin{bmatrix} 5.554\ \frac{m}{s} \\ \left(9.814 \cdot 10^{-4}\right)\ \frac{m^3}{s} \\ \left(3.833 \cdot 10^{5}\right)\ Pa \\ 8.303 \cdot 10^4 \\ 0.019 \end{bmatrix}$$

Iterative Solver Solution:

$v_{2\_sol} := \mathbf{Ans}_0 = 5.554\ \frac{m}{s}$ — *Calculated flow speed*

$V_{dot\_sol} := \mathbf{Ans}_1 = 3.533\ \frac{m^3}{hr}$ — *Calculated volume flow rate (m^3/hr and gpm shown)*

$V_{dot\_sol} = 15.555\ gpm$

$P_{loss\_sol} := \mathbf{Ans}_2 = 3.833\ bar$ — *Calculated pressure loss due to flow rate in pipe*

$Re_{D\_sol} := \mathbf{Ans}_3 = 8.303 \cdot 10^4$ — *Calculated flow Reynold's number*

$f_{D\_sol} := \mathbf{Ans}_4 = 0.019$ — *Calculated Darcy friction coefficient from Fanning friction factor in Blasius Eqn.*