

## Calculating flow out of an open pipe using Bernoulli, Darcy-Weisbach, Blasius

Bernoulli's Equation, solve for  $v_2$  (ignoring height change component)

$$P_1 + \frac{1}{2} \cdot \rho_{fluid} \cdot v_1^2 = (P_2 - \Delta P) + \frac{1}{2} \cdot \rho_{fluid} \cdot v_2^2 \xrightarrow[\text{assume, } \rho_{fluid} > 0]{\text{solve, } v_2} \left[ \frac{\sqrt{\rho_{fluid} \cdot v_1^2 + 2 \cdot \Delta P + 2 \cdot P_1 - 2 \cdot P_2}}{\sqrt{\rho_{fluid}}} \right]$$

Darcy-Weisbach equation, solve for  $\Delta P$ :

$$\frac{\Delta P}{L_{pipe}} = f_D \cdot \frac{\rho_{fluid}}{2} \cdot \frac{v_{avg}^2}{D_{pipe}} \xrightarrow{\text{solve, } \Delta P} \frac{f_D \cdot v_{avg}^2 \cdot L_{pipe} \cdot \rho_{fluid}}{2 \cdot D_{pipe}}$$

Blasius equation for Fanning friction factor ( $f$ ), curve fit for smooth pipe  $2100 < Re < 100000$

$$D_{pipe} := 15 \text{ mm}$$

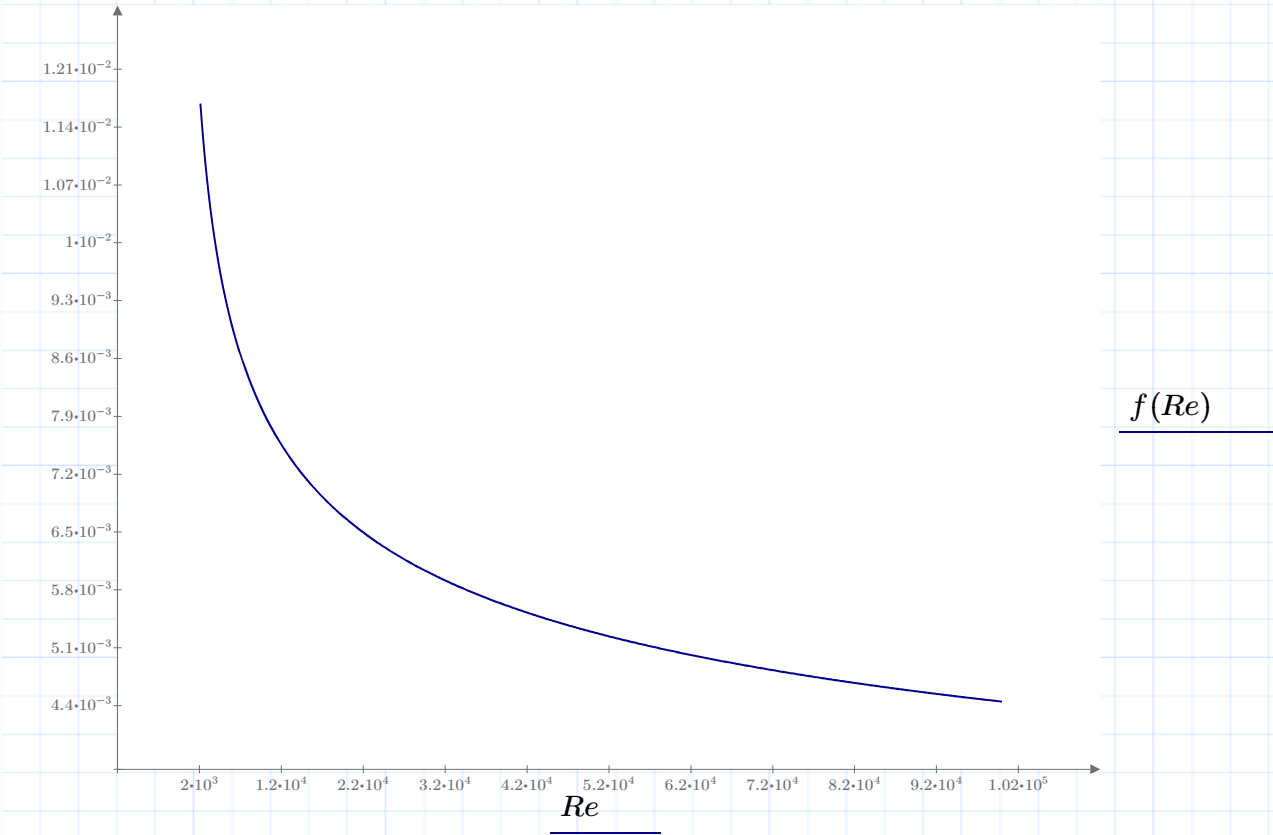
$$\nu_{water} := 1.08 \cdot 10^{-5} \frac{ft^2}{s} = (1.003 \cdot 10^{-6}) \frac{m^2}{s}$$

$$f(Re) := \frac{0.0791}{Re^{0.25}}$$

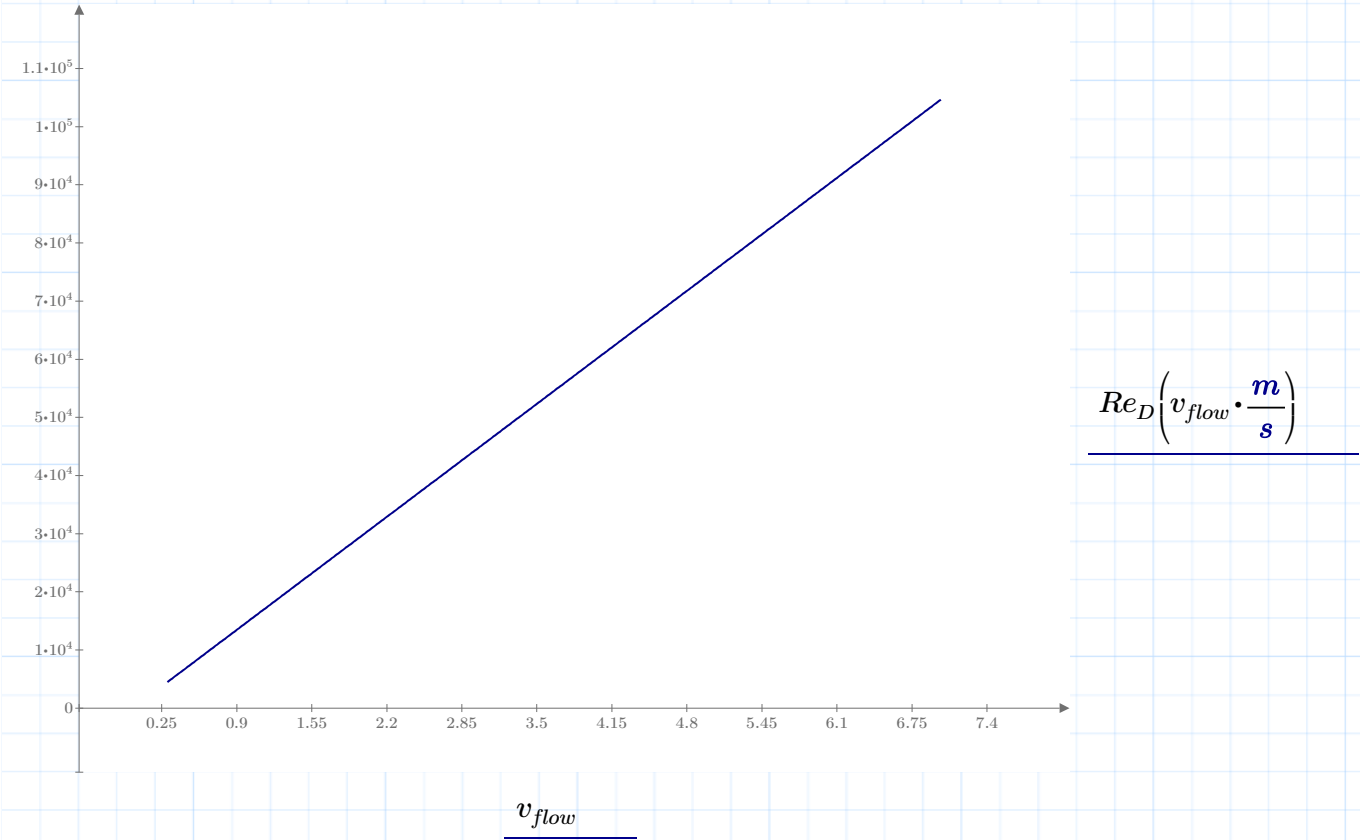
Blasius equation calculates the Fanning friction factor  $f$ . The Darcy friction factor  $f_D$  is 4x the Fanning friction factor.

$$Re_D(v_{avg}) := \frac{v_{avg} \cdot D_{pipe}}{\nu_{water}}$$

Friction Coefficient vs Reynold's Number, Blasius Equation



Flow Velocity vs. Reynold's Number, Using Pipe Dimensions



### Pipe flow parameters:

$$L_{pipe} := 20 \text{ m}$$

$$\nu_{water} := 1.08 \cdot 10^{-5} \frac{ft^2}{s} = (1.003 \cdot 10^{-6}) \frac{m^2}{s}$$

$$\rho_{water} := 1000 \frac{kg}{m^3} \quad \text{Water Density}$$

$$v_1 := 0 \frac{m}{s} \quad \text{Initial Velocity}$$

$$P_1 := 5 \text{ bar} \quad \text{Pressure at start of pipe}$$

$$P_2 := 0 \text{ bar} \quad \text{Atmospheric Pressure}$$

$$D_{pipe} = 15 \text{ mm} \quad \text{Pipe Diameter}$$

$$A_{pipe} := \frac{\pi \cdot D_{pipe}^2}{4} = 176.715 \text{ mm}^2 \quad \text{Calculate Pipe Exit Area}$$

Iterative Solve Block to find flow out of pipe (balance between flow rate and pressure loss due to flow in pipe):

Guess Values

$$v_2 := 1 \frac{m}{s}$$

$$V_{dot} := 1 \frac{m^3}{hr}$$

$$P_{loss} := 1 \text{ bar}$$

$$Re_D := 8.5 \cdot 10^4$$

$$f_D := 0.018$$

*Guess values for iterative solver, numeric solutions shown below.*

Constraints

$$(P_1 - P_{loss}) = P_2 + \frac{1}{2} \cdot \rho_{water} \cdot v_2^2$$

Bernoulli eqn., calculating flow from pressure difference. Model assumes  $P_1$  pressure is reduced by  $P_{loss}$  due to flow friction

$$\frac{P_{loss}}{L_{pipe}} = f_D \cdot \frac{\rho_{water}}{2} \cdot \frac{v_2^2}{D_{pipe}}$$

Darcy-Weisbach Eqn.

$$f_D = 4 \cdot \frac{0.0791}{Re_D^{0.25}}$$

Blasius Eqn. for estimating  $f$  from  $Re_D$ . Converted to Darcy factor using factor of 4.

$$Re_D = \frac{v_2 \cdot D_{pipe}}{\nu_{water}}$$

$Re_D$  calculation for pipe flow

$$V_{dot} = A_{pipe} \cdot v_2$$

Volume flow calculation for pipe flow

Solver

$$\text{Ans} := \text{find} (v_2, V_{dot}, P_{loss}, Re_D, f_D) = \begin{bmatrix} 6.316 \frac{m}{s} \\ 0.001 \frac{m^3}{s} \\ (4.801 \cdot 10^5) \text{ Pa} \\ 9.443 \cdot 10^4 \\ 0.018 \end{bmatrix}$$

### Iterative Solver Solution:

$$v_{2\_sol} := \text{Ans}_0 = 6.316 \frac{m}{s}$$

*Calculated flow speed*

$$V_{dot\_sol} := \text{Ans}_1 = 4.018 \frac{m^3}{hr}$$

*Calculated volume flow rate  
(m<sup>3</sup>/hr and gpm shown)*

$$V_{dot\_sol} = 17.692 \text{ gpm}$$

$$P_{loss\_sol} := \text{Ans}_2 = 4.801 \text{ bar}$$

*Calculated pressure loss due to flow rate in pipe*

$$Re_{D\_sol} := \text{Ans}_3 = 9.443 \cdot 10^4$$

*Calculated flow Reynold's number*

$$f_{D\_sol} := \text{Ans}_4 = 0.018$$

*Calculated Darcy friction coefficient from Fanning friction factor in Blasius Eqn.*