

Problem:

The approximate time to freeze standing water in an 18-inch steel pipe.

Parameters and assumptions:

The pipe is 18-inch (0.4572 m) carbon steel

Infinitely long pipe

No insulation around pipe

The pipe is exposed to the air and not buried

The pipe is completely filled with water

The water is not flowing in the pipe

The standing water is initially around $68^\circ\text{F} \Rightarrow 291.5\text{ K}$

The outside temp is around $0^\circ\text{F} \Rightarrow 255.4\text{ K}$

The freezing temp of water is $32^\circ\text{F} \Rightarrow 273.2\text{ K}$

There is a constant breeze over the pipe around $5 \frac{\text{miles}}{\text{hour}} \Rightarrow 2.24 \frac{\text{meters}}{\text{sec}}$

The convective surface heat transfer coefficient (h) at the wind/pipe interface was found by:

Calculating the Reynolds number (R_e) of the wind/pipe interface:

$$R_e = \frac{V_{air} \cdot D_o}{\nu_{air}}, \quad \text{Where : } V_{air} = \text{velocity of air} = 2.24 \frac{\text{m}}{\text{s}}$$

$$\nu_{air} = \text{Kinematic viscosity air} = 0.0000118 \frac{\text{m}^2}{\text{s}}$$

$$D_o = \text{outer diameter of pipe} = 0.4572\text{ m}$$

$$R_e = 86604.53$$

Found the Prandtl number (P_r) for air at 255.4 K $P_r = 0.720$

Calculated the Nusselt number (N_u) by $N_u = 0.023 \cdot R_e^{\frac{4}{3}} \cdot P_r^{\frac{1}{3}} = 215.695$

Found the thermal conductivity of the air $k_f = 0.02265 \frac{\text{W}}{\text{m} \cdot \text{K}}$

Calculated the surface heat transfer coefficient (h):

$$h = \frac{N_u \cdot k_f}{D_o} \approx 10.5 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

To calculate the "Overall" heat transfer coefficient (U):

$$U = \frac{1}{\left(\frac{1}{h \cdot 2 \cdot \pi \cdot r_o} + \frac{\ln \frac{r_i}{r_o}}{2 \cdot \pi \cdot k_s} \right)}$$

Where:

$$h = \text{surface heat transfer coefficient} = 10.5 \frac{W}{m^2 \cdot K}$$

$$r_o = \text{outer radius of pipe} = 0.2286 m$$

$$r_i = \text{inner radius of pipe} = 0.2191 m$$

$$k_s = \text{thermal conductivity of the steel pipe} = 60.5 \frac{W}{m \cdot K}$$

$$U = 15.375 \frac{W}{m^2 \cdot K}$$

To calculate the heat loss ($q_l = \frac{W}{m}$) per length of pipe:

$$q_l = U \cdot A_s \cdot (T_i - T_\infty)$$

Where:

$$A_s = \text{surface area per length of pipe} = \pi \cdot D_o = 1.436 \frac{m^2}{m}$$

$$T_i = \text{initial temperature of the water} = 291.5 K$$

$$T_\infty = \text{temperature of the air} = 255.4 K$$

$$q_l = 797.45 \frac{W}{m}$$

To calculate the time (t) for the water to reach the freezing temperature (T_f):

Calculate volume of water per length of pipe ($V_l = \frac{m^3}{m}$):

$$V_l = \frac{\pi}{4} \cdot D_i^2 = 0.151 \frac{m^3}{m}, \text{ Where } D_i = \text{inner diameter of pipe} = 0.43815 m$$

Calculate mass of the water per length of pipe ($M_{lw} = \frac{kg}{m}$):

$$\text{Density of water } \rho_w = 998.5 \frac{kg}{m^3}$$

$$M_{lw} = V_l \cdot \rho_w = 150.55 \frac{kg}{m}$$

Time (t_c) for the water to reach the freezing temperature T_f :

$$t_c = \frac{c_{pw} \cdot M_{lw} \cdot (T_i - T_f)}{q_l}$$

Where:

$$c_{pw} = \text{specific heat of water @ } T_i = 4185 \frac{J}{kg \cdot K}$$

$$T_i = \text{initial temperature of the water} = 291.5 K$$

$$T_f = \text{freezing temperature of the water} = 273.2 K$$

$$q_l = \text{heat loss per length of pipe} = 797.45 \frac{W}{m}$$

$$t_c = 14485.17 s \Rightarrow 4.02 \text{ hrs}$$

Time for the water to actually freeze (t_f) once it reaches the freezing temperature T_f :

$$\text{Latent heat of fusion (} h_{fs} \text{) for water is } h_{fs} = 334000 \frac{J}{kg}$$

$$\text{Density of ice is } \rho_i = 920 \frac{kg}{m^3}$$

$$\text{Mass of ice per length of pipe is } M_{li} = V_l \cdot \rho_i = 138.72 \frac{kg}{m}$$

$$t_f = \frac{h_{fs} \cdot M_{li}}{q_l} = 58098.86 s \Rightarrow 16.14 \text{ hrs}$$

Total time to freeze the water is in the pipe:

$$t_{tot} = t_c + t_f = 4.02 + 16.14 = 20.16 \text{ hrs}$$