The stress tensor

Assume we have a continuum media (gas, fluid, elastic body) in $\mathbb{R}^3 = \{(x^1, x^2, x^3)\}$. Take any material volume V of this media.The boundary ∂V of this volume is influenced by a force $\boldsymbol{F} = F^i \boldsymbol{e}_i$. This is not a mass force this is a force that acts on the volume from the contact with outside media.

Since the volume V is arbitrary it is reasonable to express the force in terms of density:

$$F^n = \int_{\partial V} \sum_{i<j} \omega^n_{ij}(x) dx^i \wedge dx^j, \quad \omega^n_{ij} = -\omega^n_{ji}. \tag{1}$$

The tensor ω has 9 independent components. I believe that these components correspond to the components of the stress tensor σ_{ij}.

By definition (formula (1) the tensor ω is defined up to addition of a tensor

$$\tilde{\omega}^n = \sum_{i<j} \tilde{\omega}^n_{ij}(x) dx^i \wedge dx^j, \quad d\tilde{\omega}^n = 0.$$

On the other hand all the equations of motion I know contain the tensor σ_{ij} only as $\nabla \sigma_{ij}$.