

• an inertial observer measures the distance and time between two events to be  $10^8$  m and 1 sec. respectively, Find the proper time between these two events.

$$\Rightarrow \text{let: } \begin{aligned} \Delta t &= 1 \\ \Delta x &= 10^8 \\ \Delta t' &= ? \\ v &= ?? \\ \Delta x' &= 0 \rightarrow \text{gives us } \Delta t' \text{ is proper time} \end{aligned}$$

These are all the relevant equations:

$$\begin{aligned} (1) \quad \Delta x' &= \gamma [\Delta x - v \Delta t] \\ (2) \quad \Delta t' &= \gamma [\Delta t - \frac{v \Delta x}{c^2}] \\ (3) \quad \Delta x &= \gamma [\Delta x' + v \Delta t'] \\ (4) \quad \Delta t &= \gamma [\Delta t' + \frac{v \Delta x'}{c^2}] \\ (5) \quad \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}$$

First we need to determine the velocity:

$$(1) \Rightarrow 0 = \frac{10^8 - v}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow \text{cannot be zero; thus } \Rightarrow \boxed{v = 10^8 \text{ m/s}}$$

from (1) we got  $v$  now calculate  $\gamma = 1.06$   
using (2)  $1 = 1.06 \Delta t'$

$$\Delta t' = 0.94 \leftarrow \text{this is the proper time}$$

$$\tau = 0.94 \text{ seconds.}$$