Problem 3

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$\vec{A} = \hat{i}\, xy + \hat{j}\,(2y - z^2) + \hat{k}\, xz$ Transform eq.: $x = r\cos\theta\sin\phi$ | $y = r\sin\theta\sin\phi$ | $z = r\cos\theta$

Matrix of position vector transform

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\theta \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\theta \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{\phi} \end{bmatrix}$$

Therefore:

$\hat{i} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\theta\,\hat{\phi}$

$\hat{j} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\theta\,\hat{\phi}$

$\hat{k} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

Plug in for x, y, z using transforms

$$\vec{A} = \hat{i}\,(r\cos\theta\sin\phi)(r\sin\theta\sin\phi) + \hat{j}\,(2r\sin\theta\sin\phi - r^2\cos^2\theta) + \hat{k}\,(r\cos\theta\sin\phi)(r\cos\theta)$$

$$\vec{A} = (r^2\sin\theta\cos\theta\sin^2\phi)\,\hat{i} + (2r\sin\theta\sin\phi - r^2\cos^2\theta) + (r^2\cos^2\theta\sin\phi)\,\hat{k}$$

unit vector transform:

replacing $\hat{i}$

$$\vec{A} = (r^2\sin\theta\cos\theta\sin^2\phi)(\sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\theta\,\hat{\phi})$$

$$= [r^2\sin^2\theta\cos\theta\sin^2\phi\cos\phi\,\hat{r} + r^2\sin\theta\cos^2\theta\sin^2\phi\cos\phi\,\hat{\theta} - r^2\sin^2\theta\cos\theta\sin^2\phi\,\hat{\phi}]$$

replacing $\hat{j}$

$$\vec{A} = (2r\sin\theta\sin\phi - r^2\cos^2\theta)(\sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\theta\,\hat{\phi})$$

$$= (2r\sin^2\theta\sin^2\phi\,\hat{r} - r^2\sin\theta\cos^2\theta\sin\phi\,\hat{r} + 2r\sin\theta\cos\theta\sin^2\phi\,\hat{\theta} - r^2\cos^3\theta\sin\phi\,\hat{\theta} + 2r\sin\theta\cos\theta\sin\phi\,\hat{\phi} - r^2\cos^3\theta\,\hat{\phi})$$

replacing $\hat{k}$

$$A = (\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})(r^2\cos^2\theta\sin\phi)$$

$$= [r^2\cos^3\theta\sin\phi\,\hat{r} - r^2\sin\theta\cos^2\theta\sin\phi\,\hat{\theta}]$$

Problem 3 cont.

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Combining Terms

$$\vec{A} = \hat{r}\left(r^2\sin^2\theta\cos\theta\sin^2\phi\cos\phi + 2r\sin^2\theta\sin^2\phi - r^2\sin\theta\cos^2\theta\sin\phi + r^2\cos^3\theta\sin\phi\right)$$

$$+ \hat{\theta}\left(r^2\sin\theta\cos^2\theta\sin^2\phi\cos\phi + 2r\sin\theta\cos\theta\sin^2\phi - r^2\cos^3\theta\sin\phi - r^2\sin\theta\cos^2\theta\sin\phi\right)$$

$$+ \hat{\phi}\left(-r^2\sin^2\theta\cos\theta\sin^2\phi + 2r\sin\theta\cos\theta\sin\phi - r^2\cos^3\theta\right) - 0$$

The total transform