

Let  $Y = \alpha \tan \pi X$ , where  $X$  is uniformly distributed in the interval  $(-1, 1)$ .

- a) Show that  $Y$  is a Cauchy random variable
- b) Find the pdf of  $Y = 1/X$

My problem is with the interval. I don't have a problem with the first part of the question.

Relevant Equations:

$$f_Y(y) = \frac{f_X(x^{(1)})}{|g'(x^{(1)})|}$$

$$\text{Cauchy: } f_X(x) = \frac{\alpha/\pi}{\alpha^2 + x^2}$$

Here is what I was able to do

After differentiating, I got:

$$g'(x^{(1)}) = \pi \alpha \sec^2 \pi x$$

Making  $x$  the subject, I had:

$$x = \frac{1}{\pi} \tan^{-1} \left( \frac{y}{\alpha} \right)$$

$$\therefore f_Y(y) = \frac{f_X \left( \frac{1}{\pi} \tan^{-1} \frac{y}{\alpha} \right)}{|\pi \alpha \sec^2 \pi x|}$$