

$\omega_n \in \text{Reals}$

$\omega_{n'} \in \text{Reals}$

$\text{Elements}[\mathbf{q}, \mathbf{q}', \text{Reals}]$

$\xi[\mathbf{q}_-] := \mathbf{q}^2 / 2$

$$f1 = \frac{1}{(\dot{\mathbf{i}} \omega_n + \xi[\mathbf{k}] - \xi[\mathbf{k} - \mathbf{q}']) (\dot{\mathbf{i}} \omega_n + \dot{\mathbf{i}} \omega_{n'} + \xi[\mathbf{k}] - \xi[\mathbf{k} - \mathbf{q} - \mathbf{q}'])}$$

$$\int_{-1}^1 f1 \, d\mathbf{k}$$

Solution Given :

$$\text{If} \left[\frac{\text{Im}[\mathbf{q}']^2 + \frac{\sqrt{\text{Im}[\mathbf{q}']^5 - 16 \text{Im}[\omega_{n'}] \text{Re}[\omega_{n'}] \text{Re}[\mathbf{q}'] + 2 \text{Im}[\mathbf{q}']^3 \text{Re}[\mathbf{q}']^2 + \text{Im}[\mathbf{q}'] \text{Re}[\mathbf{q}']^4}}{\sqrt{\text{Im}[\mathbf{q}']}}}{\text{Re}[\mathbf{q}']} \geq 4 + 3 \text{Re}[\mathbf{q}'] \mid \mid \right.$$

$$\frac{\text{Im}[\mathbf{q}']^2 + 4 \text{Re}[\mathbf{q}'] + \frac{\sqrt{\text{Im}[\mathbf{q}']^5 - 16 \text{Im}[\omega_{n'}] \text{Re}[\omega_{n'}] \text{Re}[\mathbf{q}'] + 2 \text{Im}[\mathbf{q}']^3 \text{Re}[\mathbf{q}']^2 + \text{Im}[\mathbf{q}'] \text{Re}[\mathbf{q}']^4}}{\sqrt{\text{Im}[\mathbf{q}']}}}{\text{Re}[\mathbf{q}']} \leq 3 \text{Re}[\mathbf{q}'] \mid \mid$$

$$8 \text{Im}[\omega_{n'}]^2 + \frac{8 \text{Im}[\omega_{n'}] \text{Re}[\omega_{n'}] \text{Re}[\mathbf{q}']}{\text{Im}[\mathbf{q}']} + \frac{1}{\sqrt{\text{Im}[\mathbf{q}']} \text{Re}[\mathbf{q}']^2} (\text{Im}[\mathbf{q}']^2 + \text{Re}[\mathbf{q}']^2) (\text{Im}[\mathbf{q}']^{9/2} +$$

$$\text{Im}[\mathbf{q}']^2 \sqrt{(\text{Im}[\mathbf{q}']^5 - 16 \text{Im}[\omega_{n'}] \text{Re}[\omega_{n'}] \text{Re}[\mathbf{q}'] + 2 \text{Im}[\mathbf{q}']^3 \text{Re}[\mathbf{q}']^2 + \text{Im}[\mathbf{q}'] \text{Re}[\mathbf{q}']^4)} +$$

$$\text{Re}[\mathbf{q}']^2 \sqrt{(\text{Im}[\mathbf{q}']^5 - 16 \text{Im}[\omega_{n'}] \text{Re}[\omega_{n'}] \text{Re}[\mathbf{q}'] + 2 \text{Im}[\mathbf{q}']^3 \text{Re}[\mathbf{q}']^2 + \text{Im}[\mathbf{q}'] \text{Re}[\mathbf{q}']^4)}) \leq$$

$$\left. 8 \text{Re}[\omega_{n'}]^2 + \frac{8 \text{Im}[\omega_{n'}] \text{Im}[\mathbf{q}'] \text{Re}[\omega_{n'}]}{\text{Re}[\mathbf{q}']} + \text{Im}[\mathbf{q}']^2 \text{Re}[\mathbf{q}']^2 + \text{Re}[\mathbf{q}']^4 \right] \&\&$$

$$(\mathbf{q}' \notin \text{Reals} \mid \mid \text{Re}[\mathbf{q}'] \leq -2 \mid \mid \text{Re}[\mathbf{q}'] \geq 2) \&\& \left(4 + 3 \text{Re}[\mathbf{q}'] + \right.$$

$$\frac{\sqrt{\text{Im}[\mathbf{q}']^5 - 16 \text{Im}[\omega_{n'}] \text{Re}[\omega_{n'}] \text{Re}[\mathbf{q}'] + 2 \text{Im}[\mathbf{q}']^3 \text{Re}[\mathbf{q}']^2 + \text{Im}[\mathbf{q}'] \text{Re}[\mathbf{q}']^4}}{\sqrt{\text{Im}[\mathbf{q}']} \text{Re}[\mathbf{q}']} \leq \frac{\text{Im}[\mathbf{q}']^2}{\text{Re}[\mathbf{q}']} \mid \mid$$

$$\begin{aligned}
& 3 \operatorname{Re}[\mathbf{q}'] + \frac{\sqrt{\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4}}{\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']}} \geq \\
& 4 + \frac{\operatorname{Im}[\mathbf{q}']^2}{\operatorname{Re}[\mathbf{q}']} \mid \mid 8 \operatorname{Re}[\omega_{n'}]^2 + \frac{8 \operatorname{Im}[\omega_{n'}] \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\omega_{n'}]}{\operatorname{Re}[\mathbf{q}']} + \\
& \frac{1}{\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']^2} \left(\operatorname{Im}[\mathbf{q}']^2 + \operatorname{Re}[\mathbf{q}']^2 \right) \left(\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']^4 + \right. \\
& \left. \operatorname{Im}[\mathbf{q}']^2 \sqrt{\left(\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4 \right)} + \right. \\
& \left. \operatorname{Re}[\mathbf{q}']^2 \sqrt{\left(\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4 \right)} \right) \geq \\
& \left. 8 \operatorname{Im}[\omega_{n'}]^2 + \operatorname{Im}[\mathbf{q}']^4 + \frac{\operatorname{Im}[\mathbf{q}']^6}{\operatorname{Re}[\mathbf{q}']^2} + \frac{8 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}']}{\operatorname{Im}[\mathbf{q}']} \right), \\
& \frac{1}{4 \left(2 \operatorname{Im}[\mathbf{q}'] \omega_{n'} + \mathbf{q}^2 \mathbf{q}' - 2 \operatorname{Im}[\omega_{n'}] \mathbf{q}' + \mathbf{q} (\mathbf{q}')^2 \right)} \left(-2 \operatorname{Im}[\mathbf{q}'] \operatorname{ArcTan}\left[\frac{2 \omega_{n'}}{(-2 + \mathbf{q}') \mathbf{q}'} \right] + 2 \operatorname{Im}[\mathbf{q}'] \operatorname{ArcTan}\left[\frac{2 \omega_{n'}}{\mathbf{q}' (2 + \mathbf{q}')} \right] + \right. \\
& \left. 2 \operatorname{Im}[\mathbf{q}'] \operatorname{ArcTan}\left[\frac{2 (\omega_{n'} + \omega_{n'})}{(-2 + \mathbf{q} + \mathbf{q}') (\mathbf{q} + \mathbf{q}')} \right] - 2 \operatorname{Im}[\mathbf{q}'] \operatorname{ArcTan}\left[\frac{2 (\omega_{n'} + \omega_{n'})}{(\mathbf{q} + \mathbf{q}') (2 + \mathbf{q} + \mathbf{q}')} \right] + \right. \\
& \left. \operatorname{Log}\left[4 \omega_{n'}^2 + (-2 + \mathbf{q}')^2 (\mathbf{q}')^2 \right] - \operatorname{Log}\left[4 \omega_{n'}^2 + (\mathbf{q}')^2 (2 + \mathbf{q}')^2 \right] - \right. \\
& \left. \operatorname{Log}\left[4 \omega_{n'}^2 + 8 \omega_{n'} \omega_{n'} + 4 \omega_{n'}^2 + \left((-2 + \mathbf{q}) \mathbf{q} + 2 (-1 + \mathbf{q}) \mathbf{q}' + (\mathbf{q}')^2 \right)^2 \right] + \right. \\
& \left. \operatorname{Log}\left[4 \omega_{n'}^2 + 8 \omega_{n'} \omega_{n'} + 4 \omega_{n'}^2 + \left(\mathbf{q} (2 + \mathbf{q}) + 2 (1 + \mathbf{q}) \mathbf{q}' + (\mathbf{q}')^2 \right)^2 \right] \right), \\
& \operatorname{Integrate}\left[\frac{1}{(2 \omega_{n'} - \operatorname{Im}[\mathbf{q}'] (2 \mathbf{k} - \mathbf{q}') \mathbf{q}') (2 \omega_{n'} - \operatorname{Im}[\mathbf{q}'] (2 \operatorname{Im}[\omega_{n'}] + (2 \mathbf{k} - \mathbf{q} - \mathbf{q}') (\mathbf{q} + \mathbf{q}'))} \right], \\
& \{\mathbf{k}, -1, 1\}, \text{Assumptions} \rightarrow \\
& \left(\left(\frac{\operatorname{Im}[\mathbf{q}']^2 + \frac{\sqrt{\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4}}{\sqrt{\operatorname{Im}[\mathbf{q}']}}}{\operatorname{Re}[\mathbf{q}']}} \geq 4 + 3 \operatorname{Re}[\mathbf{q}'] \mid \mid \right. \\
& \left. \frac{1}{\operatorname{Re}[\mathbf{q}']} \left(\operatorname{Im}[\mathbf{q}']^2 + 4 \operatorname{Re}[\mathbf{q}'] + \frac{1}{\sqrt{\operatorname{Im}[\mathbf{q}']}} \left(\sqrt{\left(\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4 \right)} \right. \right. \right. \\
& \left. \left. \left. \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4 \right) \right) \right) \leq 3 \operatorname{Re}[\mathbf{q}'] \mid \mid 8 \operatorname{Im}[\omega_{n'}]^2 + \frac{8 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}']}{\operatorname{Im}[\mathbf{q}']} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']^2} \left(\operatorname{Im}[\mathbf{q}']^2 + \operatorname{Re}[\mathbf{q}']^2 \right) \left(\operatorname{Im}[\mathbf{q}']^{9/2} + \operatorname{Im}[\mathbf{q}']^2 \sqrt{\operatorname{Im}[\mathbf{q}']^5 -} \right. \\
& \quad \left. 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4 \right) + \operatorname{Re}[\mathbf{q}']^2 \\
& \quad \sqrt{\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4} \Big) \leq \\
& \quad \left. 8 \operatorname{Re}[\omega_{n'}]^2 + \frac{8 \operatorname{Im}[\omega_{n'}] \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\omega_{n'}]}{\operatorname{Re}[\mathbf{q}']} + \operatorname{Im}[\mathbf{q}']^2 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Re}[\mathbf{q}']^4 \right) \& \\
& (\mathbf{q}' \notin \text{Reals} \mid \mid \operatorname{Re}[\mathbf{q}'] \leq -2 \mid \mid \operatorname{Re}[\mathbf{q}'] \geq 2) \& \left(4 + 3 \operatorname{Re}[\mathbf{q}'] + \frac{1}{\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']} \right. \\
& \quad \left. \left(\sqrt{\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4} \right) \leq \right. \\
& \quad \frac{\operatorname{Im}[\mathbf{q}']^2}{\operatorname{Re}[\mathbf{q}']} \mid \mid 3 \operatorname{Re}[\mathbf{q}'] + \frac{1}{\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']} \left(\sqrt{\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2} \right. \\
& \quad \left. \left. \left. \left. \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4 \right) \right) \geq 4 + \frac{\operatorname{Im}[\mathbf{q}']^2}{\operatorname{Re}[\mathbf{q}']} \mid \mid 8 \operatorname{Re}[\omega_{n'}]^2 + \frac{8 \operatorname{Im}[\omega_{n'}] \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\omega_{n'}]}{\operatorname{Re}[\mathbf{q}']} + \right. \\
& \quad \frac{1}{\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']^2} \left(\operatorname{Im}[\mathbf{q}']^2 + \operatorname{Re}[\mathbf{q}']^2 \right) \left(\sqrt{\operatorname{Im}[\mathbf{q}']} \operatorname{Re}[\mathbf{q}']^4 + \operatorname{Im}[\mathbf{q}']^2 \sqrt{\operatorname{Im}[\mathbf{q}']^5 -} \right. \\
& \quad \left. 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4 \right) + \operatorname{Re}[\mathbf{q}']^2 \\
& \quad \left. \sqrt{\operatorname{Im}[\mathbf{q}']^5 - 16 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}'] + 2 \operatorname{Im}[\mathbf{q}']^3 \operatorname{Re}[\mathbf{q}']^2 + \operatorname{Im}[\mathbf{q}'] \operatorname{Re}[\mathbf{q}']^4} \right) \geq \\
& \quad \left. 8 \operatorname{Im}[\omega_{n'}]^2 + \operatorname{Im}[\mathbf{q}']^4 + \frac{\operatorname{Im}[\mathbf{q}']^6}{\operatorname{Re}[\mathbf{q}']^2} + \frac{8 \operatorname{Im}[\omega_{n'}] \operatorname{Re}[\omega_{n'}] \operatorname{Re}[\mathbf{q}']}{\operatorname{Im}[\mathbf{q}']} \right) \Big] \Big]
\end{aligned}$$