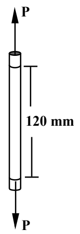
PROBLEM 1

A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a 15 mm diameter rod, and it is subject to a 3.5 kN tensile force. Knowing that an elongation of 11 mm and a decrease in diameter of 0.62 mm are observed in a 120 mm gage length, find modulus of elasticity, modulus of rigidity and Poisson's ratio of the material



Given:
Rod diameter (d_{Rod}) =15 mm
Force **(P)** = 3.5 kN
Elongation **(δ)** = 11 mm
Decrease diameter **after elongation* **(Δd)**= -0.62 mm
Gage length (L) = 120 mm
*Area Rod = $\pi(d)^2/4$
*Area Rod = $3.1416(15\ mm)^2/4$
*Area Rod = 176.72 mm^2 or 0.1767 m^2

Find:
Modulus of elasticity ***(E)*** **= ?**
Modulus of rigidity **(G) =?**
Poisson's ratio ***(V)*** **=?**

Solution
1) Uniform units;
3.5 kN = 3500 N
15 mm = 0.015 m
120 mm = 0.12 m
11 mm = 0.011 m

2) Then using Young Modulus formula;

σ = E ∗ ε, which => **E = σ/ε** => and by definition of stress:
σ =P/A, thus;
E = (P/A) / (δ/L)**,** therefore**;**
E = (P*L) / (δ*A)
substituting numerical values;
E = [(3500 N) ∗ (0.12 m)] / [(0.011 m)∗(0.176 m^2)
E = 216942.15 N/m^2
or E = 216.94 MPa

3) Since strain is applied;
$\varepsilon' = \Delta d/d_{Rod}$

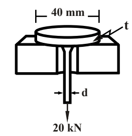
$\varepsilon' = -0.62$ mm / 15 mm => – 0.04133

4) Poisson's ratio;
V = - ε'/ ε = > –0.04133 / (δ/L)
V = -0.04133 / 0.09166
V = 0.4509

5) Modulus of rigidity;
G = E / 2(1+V)
G = 216.94 MPa / 2(1+0.4509)
G = 74.76 MPa

PROBLEM 2

The suspender rod is supported at its end by a fixed-connected circular disk as shown. If the rod passes through a 40 mm diameter hole, determine the minimum required diameter of the rod and the minimum thickness of the disk needed to support the 20 kN load. The allowable normal stress for the rod is σ_{allow} = 60 MPa, and the allowable shear stress for the disk is τ_{allow} = 35 MPa



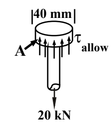
Given:
Hole diameter (d_{Hole}) = 40 mm
Load **(P)** = 20.0 kN or 20(10^3)N
Allowable normal stress for rod **(σ_{allow})** = 60 MPa or 60(10^6)N/m^2
Allowable shear stress for disk (**τ_{allow}**) =35 MPa or 35(10^6)N/m^2

Find:
Minimum required diameter of rod ***(d_{rod})*** **= ?**
Minimum thickness of disk **(*t_{disk}*) =?**

Solution
1) Area of the Rod

A = P/σ_{allow}
A = [20(10^3)N]/[60*(10^6)N/m^2]
A = 0.3333(10^{-3}) m^2

therefore;



A = π (d^2)/4
A = 0.3333(10^{-2}) m^2
d_{rod} = 0.0206 m or 20.6 mm

2) Thickness of disk

A = V/τ_{allow}
A = 20(10^3) N/ 35(10^6) N/m2
A= 0.571(10^{-3}) m2

And since the sectioned area is;
A=2π(0.02m)*(t)

then the required thickness for the disk will be:

t = 0.5714(10^{-3}) m^2/2π(0.02m)
t = 4.55(10^{-3}) m or 4.55 mm

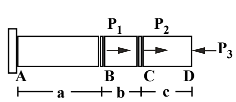
PROBLEM 3

A steel bar **AD**, has a cross-sectional area of 0.40 in^2, and is loaded by forces P_1 =2700 lb, P_2 = 1800 lb and P_3 = 1300 lb. The lengths of the segments of the bar are: a = 60.0 in, b = 24.0 in, c = 36.0 in.

(A) Assuming that the modulus of elasticity (***E***) = 30 **x** 10^6 psi, calculate the change in length **δ** of the bar

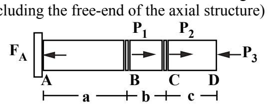
(B) By what amount P, should the load P_3 be increased? so that the bar does not change in length when the three loads are applied?

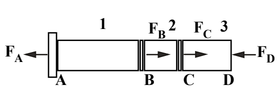
Given:
Steel rod cross-sectional area = 0.40 in2
Force (P1) = 2700 lb
Force (P2) = 1800 lb
Force (P3) = 1800 lb
Length segment a = 60 in
Length segment b = 24 in
Length segment c = 36 in
Modulus of Elasticity (***E***) = 30 **x** 10^6 psi

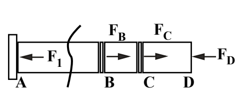


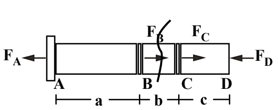
Find:
Change of length **(δ) = ?**
ΔP increasing **=?**

Solution.
To find the internal axial forces on the segments, we made 3 cuts through the bar. (including the free-end of the axial structure)







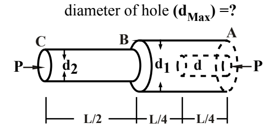


PROBLEM 4

A bar ABC of length L consists of two parts of equal lengths but different diameters
Segment AB has a diameter d_1 = 100 mm and segment BC has diameter d_2 = 60 mm.
Both segments have length L/1 = 0.6 m. A longitudinal hole of diameter d is drilled through segment AB for one-half of its length (distance L/4 = 0.3 m).
The bar is made of plastic with modulus of elasticity, E = 4.0 GPa.
The compressive load P = 110 kN.
If the shortening of the bar is limited to 8.0 mm what is the maximum allowable diameter of the hole?

Given:
Segment AB (d_1) =100 mm = 0.10 m
Segment BC (d_2) = 60 mm = 0.06 m
Length of segments AB and BC (L/2) = 0.6 m.
Length of one-half AB (L/4) = 0.3 m
Total length of bar = 1.2 m.
Modulus of Elasticity (E) = 4.0 GPa.
Force **(P)** = 110 kN or 11000 N
Maximum shortening of bar **(δ_{Max})** = 8.0 mm or 0.008 m

Find:
Maximum allowable diameter of hole (**d_{Max}**) **=?**



Shortening (δ) of the bar;

$$\Sigma = \frac{N_i \times L_i}{E_i \times A_i} \Rightarrow \frac{P}{E} \Sigma \frac{L_i}{A_i}$$

$$\Rightarrow \frac{P}{E} \left[\frac{L/\cancel{4}}{\pi/\cancel{4}(d_1^2 - d^2)} + \frac{L/\cancel{4}}{\pi/\cancel{4}(d_1^2)} + \frac{L/\cancel{2}}{\pi/\cancel{4}(d_2^2)} \right]$$

$$\Rightarrow \frac{PL}{E\pi} \left[\frac{1}{[(0.10\ m)^2 - (d)^2]} + \frac{1}{(0.10\ m)^2} + \frac{2}{(0.06\ m)^2} \right]$$

$$0.008\ m = \frac{(110000N) \times (1.2\ m)}{(4.0 * 10^9\ Pa)\pi} \left[\frac{1}{[(0.10\ m)^2 - (d)^2]} + \frac{1}{(0.10\ m)^2} + \frac{2}{(0.06\ m)^2} \right]$$

$$761.598 = \left[\frac{1}{0.010\ m^2 - (d)^2} + \frac{1}{0.010\ m^2} + \frac{2}{0.0036\ m^2} \right]$$

$$761.598 = \left[\frac{1}{0.010\ m^2 - (d)^2} + 100\ m2 + 555.555\ m2 \right]$$

$$\frac{1}{0.010\ m^2 - (d)^2} = 761.598 - 100\ m^2 - 555.555\ m^2$$

$$\frac{1}{0.010\ m^2 - (d)^2} = 106.043$$

$$\frac{1}{106.043} = 0.010\ m^2 - (d)^2$$

$d^2 = 0.009430226 - 0.010\ m^2$

$\sqrt{d^2} = \sqrt{5.6977 \times 10^{-4}\ m^2}$ $\quad d^2 = 0.023869$ m or 23.87 mm

PROBLEM 3 *continuation

Therefore by Equilibrium;

$\Sigma Fx = - F_1 + 27000 \text{ lb} + 1800 \text{ lb} + (- 1300 \text{ lb}) = 0$

F1 = (2700 lb + 1800 lb) - 1300 lb
F1 = 3200 lb (Tension)

$\Sigma Fx = - F_2 + 1800 \text{ lb} + (-1300 \text{ lb}) = 0$

F2 = 1800 lb - 1300 lb
F2 = 500 lb (Tension)

$\Sigma Fx = - F_3 - 1300 \text{ lb} = 0$

F3 = -1300 lb (Compression)

Normal Stress calculations:
Segment 1 (or AB)

$\sigma = F_1 / A_1$

$\sigma = 3200 \text{ lb} / 0.40 \text{ in}^2$

σ= 8000 psi (Tension)

Rod elongation on segment 1 (or AB)

$\delta_1 = (F_1 \times L_1)/(A_1 \times E_1)$

$$\delta_1 = \frac{(3200 \text{ lb} \times 6.0 \text{ in})}{[(0.40 \text{ in}^2) \times [(30 \times 10^6 \text{ psi})]}$$

$\delta_1 = 0.0016$ in (elongation)

Segment 2 (or BC)

$\sigma_2 = F_2 / A_2$

$\sigma_2 = 500 \text{ lb} / 0.40 \text{ in}^2$

$\sigma_2 = 1200$ psi (Tension)

Rod elongation on segment 2 (or BC)

$\delta_2 = (F_2 \times L_2)/(A_2 \times E_2)$

$$\delta_2 = \frac{(500 \text{ lb} \times 2.4 \text{ in})}{[(0.40 \text{ in}^2) \times [(30 \times 10^6 \text{ psi})]}$$

$\delta_2 = 1.000 \times 10^{-4}$ in (elongation)

Normal Stress calculations:
Segment 3 (or CD)

$\sigma_3 = F_3 / A_3$

$\sigma_3 = - 1300 \text{ lb} / 0.40 \text{ in}^2$

$\sigma_3 = -3250$ psi (Compression)

Rod elongation on segment 3 (or CD)

$\delta_3 = (F_3 \times L_3)/(A_3 \times E_3)$

$$\delta_3 = \frac{(1300 \text{ lb} \times 3.6 \text{ in})}{[(0.40 \text{ in}^2) \times (30 \times 10^6 \text{ psi})]}$$

$\delta_3 = -3.90 \times 10^{-4}$ in (contraction)

Then the total change on length of the steel rod will be;

$\delta = -3.90 \times 10^{-4} \text{ in} + 1.000 \times 10^{-4} + 0.0016 \text{ in}$

$\delta = 0.00131$ in.

Finally, to avoid any change on length, P should be equally opposite in magnitude to the total change on length of the rod.

$\delta_1 = (F \times L)/(A \times E)$

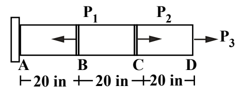
$0.0131 \text{ in} = [F \times (120 \text{ in})] / [(30 \times 10^6 \text{ psi}) \times (0.40 \text{ in}^2)]$

F = 1311 lb.

In other words the load P3 should be increased by 11 lb.

PROBLEM 3 (Variation)

A prismatic bar AD is subjected to loads P1, P2 and P3 acting at points B, C, and D respectively. Each segment of the bar is 20 in. long. The bar has a cross-sectional area A = 1.40 in² and is made of copper with E = 17000 ksi.



Given:
P1 = 3 k
P2 = 3 k
P3 = 11 k
E = 17000 ksi
A = 1.40 in²

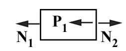
Find:

(a) Displacement (δ_D) at the free end of the bar

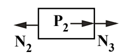
(b) What should be the load P3 if its desired to reduce the displacement at end D to half of its original value?

Solution.

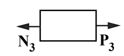
$N_1 = N_2 - P_1$
$\quad = P_2 + P_3 - P_1$



$N_2 = N_3 + P_2$
$\quad = P_2 + P_3$



$N_3 = P_3$



$$\delta_D = \frac{N_1 \times L}{E \times A} + \frac{N_2 \times L}{E \times A} + \frac{N_3 \times L}{E \times A}$$

$$\delta_D = \frac{L}{(E)(A)}[(P_2 + P_3 - P_1) + (P_2 + P_3) + P_3]$$

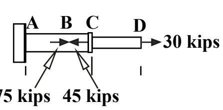
$$\delta_D = \frac{L}{(E)(A)}(2P_2 + 3P_3 - P_1)$$

$\delta_D = 0.03025$ in.

$$\frac{L}{(E)(A)}(2P_2 + 3P_3 - P_1)$$

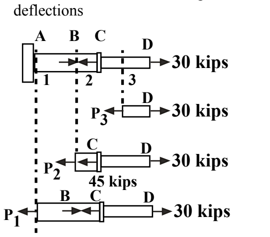
PROBLEM 4 (Variation)

Given:
$L_{AB} = 12$ in
$L_{BC} = 12$ in
$L_{CD} = 16$ in
$A_{AC} = 0.9 \text{ in}^2$
$A_{CD} = 0.3 \text{ in}^2$



Find:
Deformation of the steel rod under the given loads.

Solution:
- Divide the rod into components the load application points.
- Apply a FBD analysis on each component to find internal force.
- Evaluate the total of the component deflections



Applying FBD analysis to each component to find internal forces.

P_1 = 75 kips - 45 kips + 30 kips
P_1 = 60 kips or 60 x 10^3 lb
P_1 = 60 x (10^3) lb
P_2 = -45 kips + 30 kips
P_2 = -15 kips or 15 x 10^3 lb
P_2 = -15 x (10^3) lb
P_3 = 30 kips
P_3 = 30 x 10^3 lb
P_3 = 30 x (10^3) lb

Evaluating total deflection.

$$\delta = \Sigma_i \frac{P_i L_i}{A_i E_i} \Rightarrow \frac{1}{E}\left[\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right]$$

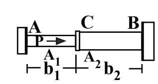
$$\delta = \frac{1}{29 \times 10^6}\left[\frac{(60 \times 10^3)12}{0.9} + \frac{(-15 \times 10^3)12}{0.9} + \frac{(30 \times 10^3)12}{0.3}\right]$$

$$\boldsymbol{\delta = 75.9 \times 10^{-3} \text{ in.}}$$

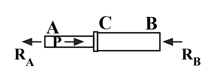
PROBLEM 4 (Variation)

A bar ABC having two different cross-sectional areas; A_1 and A_2 is held between rigid supports at A and B. A load P acts at point C, which is distance b_1 from end A and distance b_2 from end B.

Find:
a) Reactions RA and RB at supports A and B, due to the load P
b) Obtain a formula for displacement δ_c of point C
c) What is the ratio of the stress σ_1 in region AC to the stress σ_2 in region CB**?**



Solution.
Remove both supports at the end and draw a FBD of bar ABC.



Then by Equilibrium;

$\Sigma F_{Horiz} = 0 \Rightarrow R_A + R_B = P$ **(Eq. 1)**

Then by Compatibility;

δ_{AC} = elongation of segment AC

δ_{CB} = shortening of segment CB

therefore; $\delta_{AC} = \delta_{CB}$ **(Eq. 2)**

Force Displacement relations;

$$\delta_{AC} = \frac{(R_A) \times (b_1)}{(E) \times (A_1)} \quad \text{(Eq. 3)}$$

$$\delta_{CB} = \frac{(R_B) \times (b_2)}{(E) \times (A_2)} \quad \text{(Eq. 4)}$$

Equating:
(Eq. 3 and Eq. 4) into Eq. 2

$$\frac{(R_A) \times (b_1)}{(E) \times (A_1)} = \frac{(R_B) \times (b_2)}{(E) \times (A_2)} \quad \textbf{(Eq. 5)}$$

Then equating (Eq. 1 and Eq. 5)

$$R_A = \frac{(b_2) \times (A_1) P}{(b_1 \times A_2) + (b_2 \times A_1)}$$

$$R_B = \frac{(b_1) \times (A_2) P}{(b_1 \times A_2) + (b_2 \times A_1)}$$

Displacement of point C

$$\delta_C = \delta_{AC} = \frac{(b_1) \times (R_A)}{E \times A_1}$$

$$\Rightarrow \frac{[(b_1) \times (b_2)]P}{E\,[(b_1 \times A_2) + (b_2 \times A_1)]}$$

Ratio of stresses

$\sigma_1 = \dfrac{R_A}{A_1}$ **(Tension)**

$\sigma_2 = \dfrac{R_B}{A_2}$ **(Compression)**

$$\frac{\sigma_1}{\sigma_2} = \frac{b_2}{b_1}$$