

Conditional Probability for Discrete R.V.

©

Indep.

Q: How to Compute X, Y - discrete R.V.

$P\{X=k \mid X+Y=p\}$, p, k some numbers or fixed constants.

$$P\{X=k \mid X+Y=p\} = \frac{P\{X=k \text{ and } X+Y=p\}}{P\{X+Y=p\}} =$$

$$= \frac{P\{X=k \text{ and } Y=p-k\}}{P\{X+Y=p\}} =$$

i.e. event $\{X=k \text{ and } X+Y=p\}$ is the same as the event $\{X=k \text{ and } Y=p-k\}$

$$= \frac{P\{X=k\} \cdot P\{Y=p-k\}}{P\{X+Y=p\}}$$

We can split into a product b/c X and Y are indep

①.

$$P\{X=k\} = P\{Y=k\} = \begin{pmatrix} 0 & 1/3 \\ 1 & 2/3 \end{pmatrix}$$

$$P\{X=k \mid X+Y=1\} =$$

$$= \frac{P\{X=k \text{ and } Y=1-k\}}{P\{X+Y=1\}} = \text{b/c Independent.}$$

$$= \frac{P\{X=k\} \cdot P\{Y=1-k\}}{P\{X+Y=1\}}$$

$$1. P\{X+Y=1\} = P\{X=0\} P\{Y=1\} + P\{X=1\} P\{Y=0\} \\ = 2 \cdot \frac{2}{9} = \frac{4}{9}$$

$$2. P\{X=0 \mid X+Y=1\} = \frac{P\{X=0\} \cdot P\{Y=1\}}{4/9} = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{9}{4}$$

$$3. P\{X=1 \mid X+Y=1\} = \frac{P\{X=1\} \cdot P\{Y=0\}}{4/9} = \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{9}{4} = \frac{1}{2}$$

Similar for

$$P_r \{X=k \mid X+Y \geq p\} =$$

$$\frac{P_r \{X=k \text{ and } X+Y \geq p\}}{P_r \{X+Y \geq p\}} = \frac{P_r \{X=k\} \cdot P_r \{Y \geq p-k\}}{P_r \{X+Y \geq p\}}$$

Have to compute $P_r \{X+Y \geq p\}$
 $P_r \{Y \geq p-k\}$

Another Example

$$P_X(x) = \begin{cases} 1/3 & x=0 \\ 1/2 & x=1 \\ 1/6 & x=2 \end{cases}$$

$$P_Y(y) = \begin{cases} 1/3 & y=0 \\ 2/3 & y=1 \end{cases}$$

$$P\{X=k \mid X+Y > 1\} =$$

$$= \frac{P\{X=k\} \cdot P\{Y > 1-k\}}{P\{X+Y > 1\}}$$

$$P\{X+Y > 1\} = \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{1}{3} =$$

$$\begin{aligned} (X,Y) &= (1,1) \\ &\quad (2,1) \\ &\quad (2,0) \end{aligned} = \frac{2}{6} + \frac{2}{18} + \frac{1}{18} = \frac{1}{2}$$

$$P\{X=0 \mid X+Y>1\} = \frac{P\{X=0\} \cdot P\{Y>1\}}{P\{X+Y>1\}} = 0$$

$$P\{X=1 \mid X+Y>1\} = \frac{P\{X=1\} \cdot P\{Y>0\}}{P\{X+Y>1\}} =$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2}} = \frac{2}{3}$$

$$P\{X=2 \mid X+Y>1\} = \frac{P\{X=2\} \cdot P\{Y>-1\}}{P\{X+Y>1\}} =$$

$$= \frac{\frac{1}{6} \cdot 1}{\frac{1}{2}} = \frac{1}{3}$$

Note: $P\{X=k \mid X+Y>1\}$

↑ strict Inequality is important!

$$P\{X=k \mid X+Y \geq 1\} \neq P\{X=k \mid X+Y>1\}$$

in General.