Project III: Dynamic Analysis of the Internal Combustion Engine

Thermodynamics

1. The induction (intake) stroke: The inlet valve opens, and the piston travels from top dead center, TDC (state 0) to the bottom dead center, BDC (state 1). As the piston moves, low pressure forms in the cylinder, and an air-fuel mixture at the ambient temperature and suction pressure (a little less than the ambient pressure) is sucked into the cylinder.
2. The compression stroke: The inlet valve closes, and the piston travels from the BDC (state 1) to the TDC (state 2). In this process, both the air-fuel mixture's pressure and temperature increase. During the compression stroke, the piston does work on the gas in the cylinder. At some point during the compression process, the spark plug fires, ignition occurs, and the fuel combusts, raising both the temperature and the pressure of the gas even further (state 2 to state 3).
3. The expansion (power) stroke: The piston moves towards BDC (state 4) while the combustion process continues. The gases push the piston. Towards the end of the power stroke, the exhaust valve opens and the combustion products start escaping.
4. The exhaust stroke: As the exhaust valve opens, the pressure drops (state 4 to state 1). The piston moves from the BDC to the TDC expelling the combustion products (state 1 to state 0). During the expulsion process, the combustion products are at a temperature and pressure above ambient conditions. At the end of the expulsion process, the exhaust valve closes, and we are back where we started.

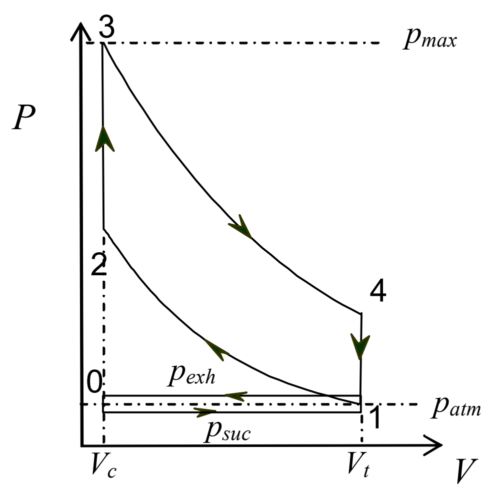

Figure 1 The P-V diagram for the ideal air cycle for the four stroke internal combustion engine.

Process 1-2: The piston moves from BDC (point 1) to TDC, compressing the gas in the cylinder. No heat exchange occurs between the gas and its surroundings during the compression process. The process is *adiabatic*. The ratio of the gas' volume at the beginning and the end of the compression process is known as the compression ratio,

$$r = \frac{V_1}{V_2} = \frac{V_4}{V_3}.$$

The adiabatic process is modeled by the equation:

$$PV^k = constant$$

Or

$$P_1 V_1^{\,k} = P_2 V_2^{\,k} \tag{1}$$

It is usual to use the value k=1.4 for air cycles.

Process 2-3: The cylinder remains at TDC without moving. The fuel chemical energy is converted into heat energy, which is added to the gas. The heat addition is assumed to occur instantaneously. During the heat addition process, the gas' volume remains fixed while its temperature and pressure increase. In other words, this is a constant volume process.

Process 3-4: The piston moves from TDC to BDC. The gas expands while pushing the piston. The expansion process is assumed to be reversible and adiabatic.

Process 4-1: The piston remains at BDC. The valves open and the pressure instantly drops to close to ambient. Heat is removed from the gas instantaneously before the cylinder has an opportunity to move. This is a constant volume process.

Some terminology is appropriate here. V_c is called the clearance volume and is denoted by V_c. Similarly, the volume of the cylinder at the bottom dead center is denoted by V_t. The difference between the two is called the *swept volume*, or the displacement, V_s. The compression ratio, r, is given by the ratio of the volume at the bottom dead center position to the clearance volume:

$$r = \frac{V_c + V_s}{V_c}\ . \tag{2}$$

Force and moment balance

Here is a brief overview of what is going on in the internal combustion engine.

1. The pressure of the gases (air + fuel, or by products of combustion) exerts a force on the piston. Think of this force as being an "input force", although during some parts of this cycle, we know that the gases in the cylinder are not doing positive work.
2. All the parts of the internal combustion engine have a finite mass (inertia). Thus a fraction of the input forces are "spent" on accelerating or decelerating the masses.
3. Some of the input is used to overcome the friction on the piston walls and the friction at the bearings. (Gravitational forces are insignificant for internal combustion engines).
4. The crank is coupled to the crankshaft which in turn is coupled via a power train to the wheels of the automobile. The crankshaft may also power the water pump, camshaft, power steering pump, the air conditioning compressor and other accessories. A significant part of the input force is used to drive the automobile. Since the crank has a rotary motion, the fraction of the input force used to drive the crank is effectively a moment, and is called the *turning moment.*

We want a formal description of these ideas and we want to develop an equation that will relate the pressure in the cylinder to the turning moment.

Basic definitions

Displacement

The swept volume of the engine.

$= n\ (V_t - V_c)$

where n = number of cylinders.

Brake horse power

The net engine output power at the crankshaft, 1 horsepower = approx. 745 Watts.

Maximum torque

The maximum average turning moment at the crankshaft.

Engine speed (RPM)

The number of revolutions per minute for the crank shaft.

Curb weight

The mass of the vehicle when ready to use (excluding the weight of the driver).

Table 1 Specifications for some commercially available automotive engines

	Audi A4, 1.8	BMW 740i	Escort 1.8	Merced es S 600	Porsche 911	Porsche 911 Turbo	Rolls-Royce	Honda Civic	Chrysler Voyager
Displacement (liters)	1.781	3.982	1.753	5.987	3.6	3.6	6.75	1.493	2.972
Brake horse power (BHP)	125	210	77	394	272	408	245	90	147
Brake horse power (kW)	93	157	57	294	203	304	183	67	110
Max Torque (Nm)	173	400	156	570	330	540	500	119	225
Engine speed for BHP peak (RPM)	3950	4500	4000	3800	6000	5750	4000	6000	5100
Curb Weight (kg)	1225	1790	1065	2180	1370	1500	2430	935	1585
Weight to power ratio (kg/kW)	13.14	11.43	18.55	7.42	6.75	4.93	13.30	13.93	14.46
Compression Ratio	10.3	10	10	10	11.3	8	8	9.2	8.9

Estimated mass and inertia

Instead of writing down the dynamic equations of motion for the system of rigid bodies (crankshaft, connecting rods and pistons), we will analyze a single slider crank mechanism, and characterize the dynamics of each rigid body by considering an equivalent system of a finite number of particles. To illustrate this, consider the link of mass m shown in Figure 2. It can be approximated by a system of two particles. If the center of mass of the system is at C, as shown in the figure, the equivalent system of a system of two particles of mass m_A and m_B is given by:

$$m_A = m\frac{b}{l}, \quad m_B = m\frac{a}{l} \tag{3}$$

where m is the total mass of the original link.

Note this is only an approximation. The equivalent system on the right in the figure has the same mass and the same location of the center of mass. However, a calculation of the mass moment of inertia about the center of mass for the two "equivalent" rigid bodies will yield different results. Nevertheless, we use this simplification in our analysis.

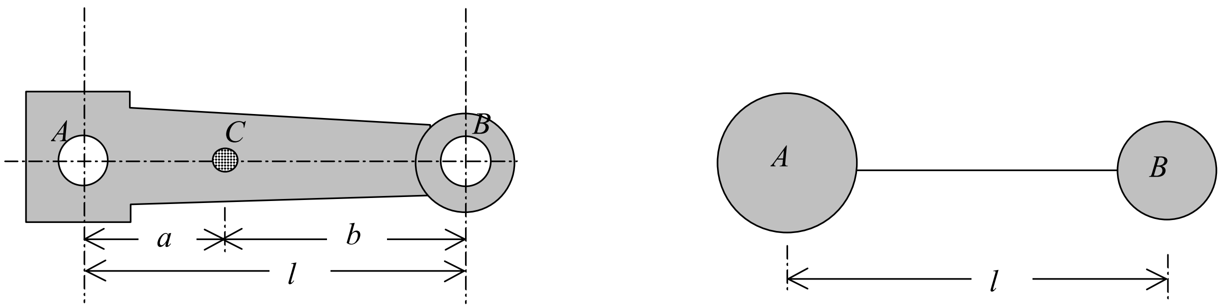

Figure 2 Any given link with the center of mass at C (see left) can be approximated by a massless rod with particles of mass m_A and m_B at A and B respectively.

Kinematics of the slider crank mechanism

We will now consider the steady-state operation of the engine at a rated RPM. In other words, we'll assume the engine is running at constant speed at a specified speed, and analyze the forces and moments that must act on the engine.

Consider the special case of the slider crank mechanism shown in Figure 3. Define the following variables:

$$\theta = \theta_2,\ \phi = 2\pi - \theta_3,\ \text{and}\ x = r_1.$$

with the lengths $r = r_2$, and $l = r_3$.

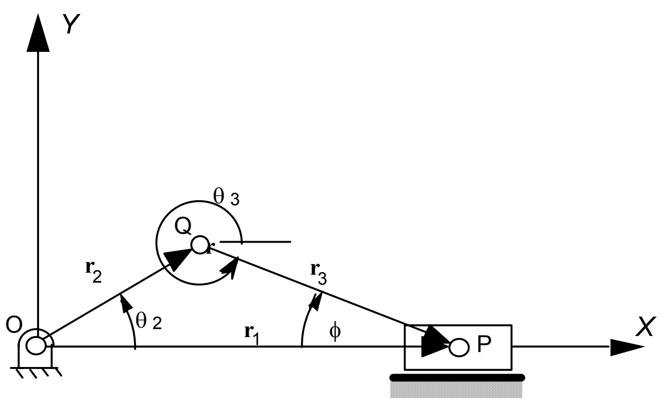

Figure 3 The slider crank mechanism in an internal combustion engine

The position closure equations are:

$$x = r\cos\theta + l\cos\phi$$

$$r\sin\theta - l\sin\phi = 0 \tag{4}$$

From these equations, substituting for θ, we can see that:

$$\phi = \sin^{-1}\left(\frac{r}{l}\sin\theta\right) \tag{5}$$

Differentiating equation (4) we get:

$$\dot{x} = -r\sin\theta\dot{\theta} - l\sin\phi\dot{\phi}$$
$$l\cos\phi\dot{\phi} - r\cos\theta\dot{\theta} = 0 \tag{6}$$

Solving the second equation above we get:

$$\dot{\phi} = \frac{r\cos\theta}{l\cos\phi}\dot{\theta} \tag{7}$$

which then yields upon substitution into the first equation in (6):

$$\dot{x} = -\frac{r\sin(\theta+\phi)}{\cos\phi}\dot{\theta} \tag{8}$$

Differentiating (7) for a constant crank velocity, we get:

$$\ddot{\phi} = -\frac{-r\sin\theta\dot{\theta}^2 + l\sin\phi\dot{\phi}^2}{l\cos\phi} \tag{9}$$

Differentiating the first equation in (6) again we get:

$$\ddot{x} = -r\cos\theta\dot{\theta}^2 - l\cos\phi\dot{\phi}^2 - l\sin\phi\ddot{\phi} \tag{10}$$

Substituting from (7) and (9) for the rates of change of ϕ we have an expression for the acceleration of the piston.

Dynamics

For the slider crank mechanism denote the masses of the crank, the piston, and the connecting rod be denoted by m_{crank}, m_{piston}, and m_{conn} respectively. Let us assume that all the rigid bodies are symmetric. In other words, the center of mass for each rigid body is at its geometric center. Following the procedure outlined above in Equation (3), we obtain the approximate model shown in Figure 4,

$$\begin{aligned} m_O &= \tfrac{1}{2}m_{crank} \\ m_Q &= \tfrac{1}{2}m_{crank} + \tfrac{1}{2}m_{conn} \\ m_P &= \tfrac{1}{2}m_{conn} + m_{piston} \end{aligned} \tag{11}$$

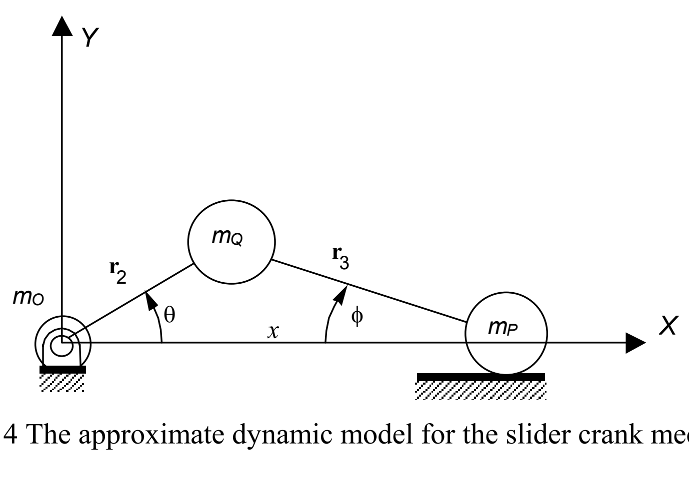

Figure 4 The approximate dynamic model for the slider crank mechanism

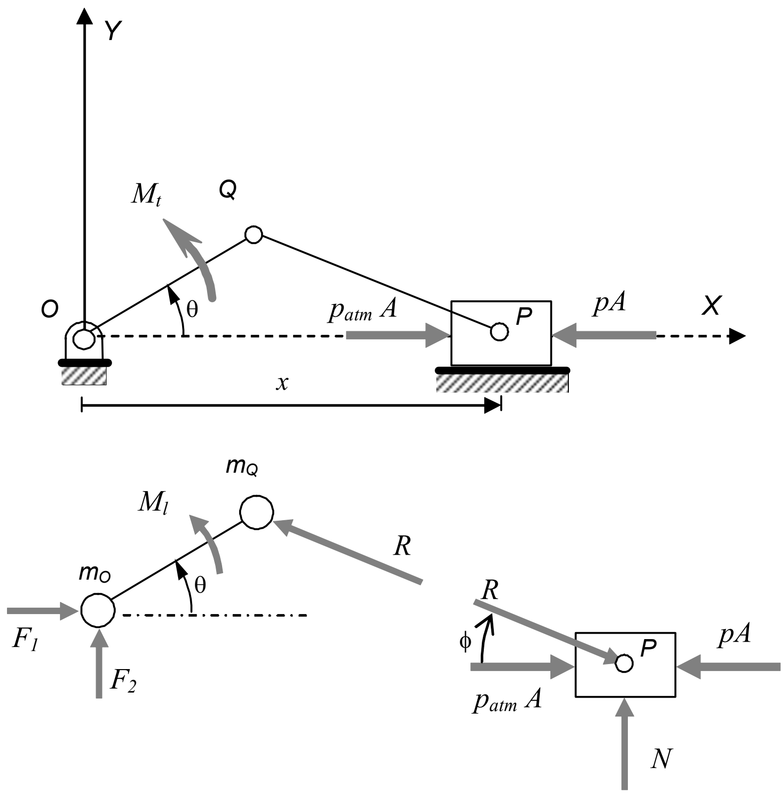

Figure 5 The external forces and moment acting on the slider crank mechanism (top) and a free body diagram of the piston and crank (bottom).

Denote the pressure in the combustion chamber by p, the load moment acting on the crankshaft by M_l, and the atmospheric pressure by p_{atm}. The reaction forces at O are F_1 and F_2 respectively, while the reaction force on the frictionless piston is N. R denotes the axial force on the massless model of the connecting rod.

Comparing the free body diagram and the inertia response diagram (not shown) of the piston, we can write the following equation in the x – direction:

$$(p_{atm} - p)A + R\cos\phi = m_P \ddot{x} \tag{12}$$

where A is the cross-sectional area of the piston given by

$$A = \frac{\pi D^2}{4},$$

D being the bore (diameter) of the piston.

Recall our assumption that the engine is running at constant speed. This is at best an approximation since the engine speed is never constant but instead fluctuates within a narrow band. However, assuming this approximation is valid (the angular acceleration of the crankshaft is zero), we can sum moments on the crankshaft about O and write:

$$rR\sin(\theta + \phi) + M_l \approx 0$$

or

$$\frac{\left[(p - p_{atm})A + m_P \ddot{x}\right]}{\cos\phi} r \sin(\theta + \phi) + M_l \approx 0 \tag{13}$$

There are two terms in this moment balance. The first term is the *turning moment*, M_t, which causes the crankshaft to turn. The second term, M_l, is the load moment which the *turning moment* must overcome in order to keep the crankshaft rotating at uniform speed. The expression for the turning moment can be broken down into two terms:

$$M_t = \frac{(p - p_{atm})Ar\sin(\theta + \phi)}{\cos\phi} + \frac{m_P \ddot{x} r \sin(\theta + \phi)}{\cos\phi}$$

The first term, denoted by M_p, is the term associated with the pressure in the combustion chamber. The second term is the inertial moment, M_i, that drives the crankshaft even when there is no combustion.

If the slider crank mechanism were mass less, the turning moment would be equal to be equal to the moment due to the pressure in the cylinder:

$$M_p = (p - p_{atm})Ar\frac{\sin(\theta + \phi)}{\cos\phi} \tag{14}$$

The moments due to the inertia of the moving parts in the mechanism is given by the inertial moment, M_i,

$$M_i = \frac{m_P \ddot{x} r \sin(\theta + \phi)}{\cos\phi} \tag{15}$$

Typical plots for the three moments are shown in Figure 6.

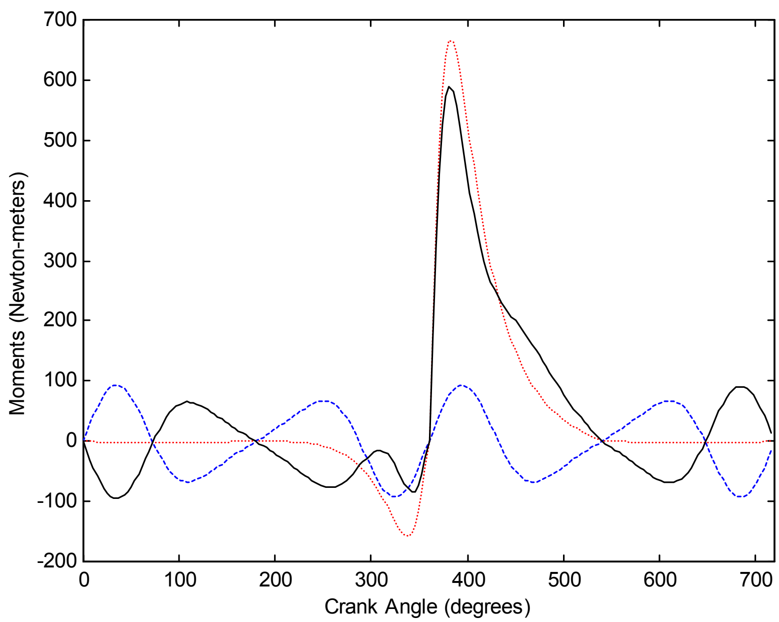

Figure 6 The moment due to the cylinder pressure (M_p, dotted), the inertial moment (M_i, dashed), and the turning moment (M_t, solid) for a single cylinder of the sample engine shown in Table 2.

Table 2 Specifications for a sample four cylinder engine

Symbol	Definition	Value
r	compression ratio	10.3
k	expansion coefficient	1.4
p_{max}	maximum pressure (after combustion), p_3	100 atmospheres
p_{atm}	atmospheric pressure	14.7 psi or 1.01325 bar
V_c	clearance volume	0.05 liters
L	stroke (2 times crank radius)	86.4 mm
D	bore	81 mm
α	ratio of crank to conn. rod length	0.35
N	Engine speed	3000 RPM
m_{piston}	mass of the piston	1 lbs
m_{crank}	effective mass of the crank shaft for one cylinder	15 lbs/4
m_{conn}	mass of the connecting rod	1.75 lbs

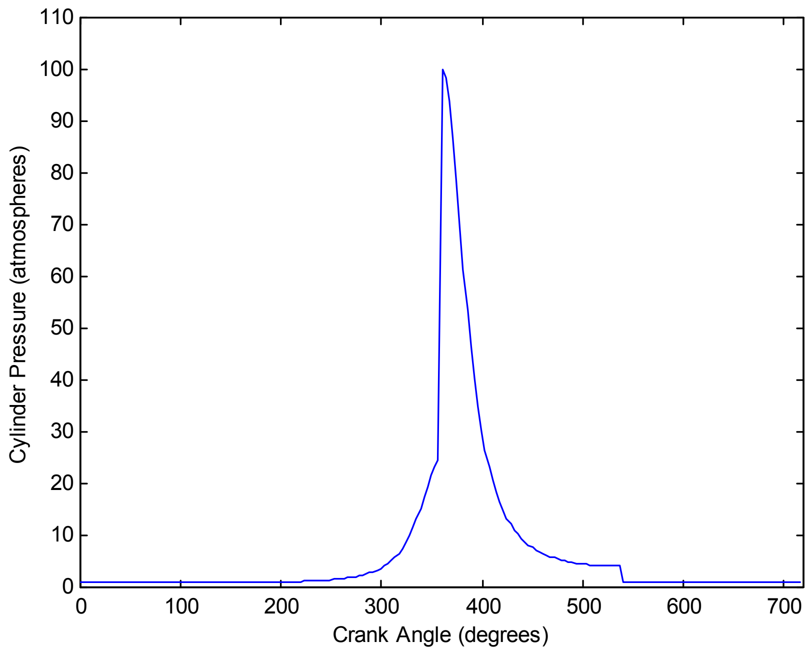

Figure 7 The cylinder pressure in atmospheres for a single cylinder of the sample engine shown in Table 2. An ideal air cycle is assumed except for two changes. The intake pressure is assumed to be 0.9 atmospheres and the exhaust pressure is assumed be 1.0 atmospheres.

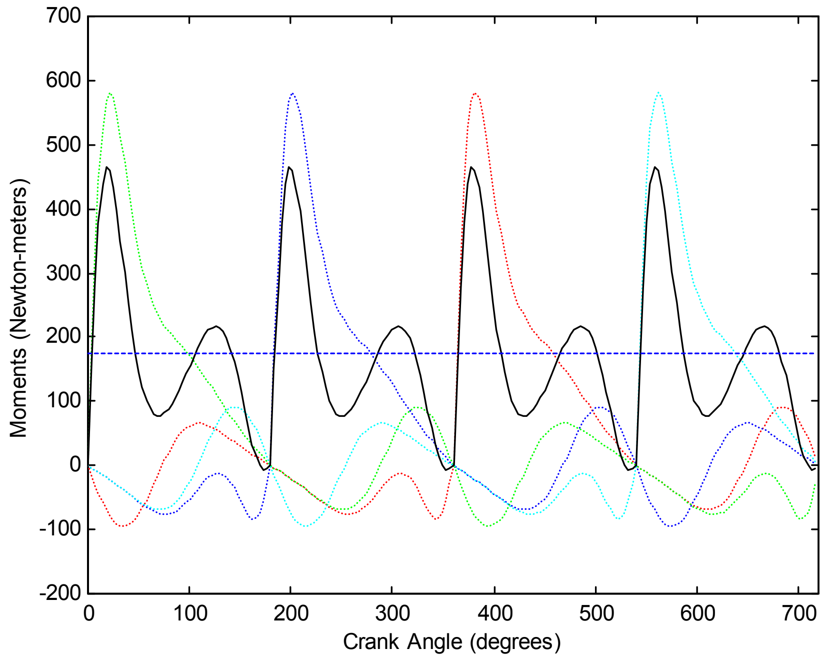

Figure 8 The turning moment for a balanced four cylinder engine. The dotted lines show the turning moments from each cylinder while the solid line shows the total turning moment. The dashed line shows the average torque at the crankshaft. The data for the plot comes from Table 2.

Exercise: Computer aided analysis of a four-stroke internal combustion engine

The goal of this exercise is to develop the mathematical model and a software package that will allow you to analyze a four-stroke internal combustion engine. Specifically we want to be able to do the following tasks for a four-stroke engine of your choice.

1. Plot the cylinder pressure against the crank angle for 720 degrees (a complete cycle). Note that Figure 1 shows the pressure against the cylinder volume – we want pressure against the crank angle.

2. Derive an expression for the piston speed as a function of crank angle and crank speed and plot the piston speed, $\dot{x}$, against the crank angle for a constant crank speed. (see Figure 3).

3. Plot the inertial moment against the crank angle

4. Plot the turning moment against the crank angle.

5. Plot the turning moment for the engine for a 720 degree rotation by overlaying the plots for a single cylinder at the appropriate intervals.

6. Calculate the average turning moment (engine torque).

7. Estimate the engine brake horse power using your calculation of the engine torque and the rated RPM.

8. Suggest a suitable remedy for reducing the engine fluctuations.

Note

You may choose any engine to analyze. The data in Table 2 is a good starting point. You can find data for other engines on the net if you go to automobile manufacturer's websites. However, there are four variables that you will need to fix for yourself.

- You will need to increase the masses m_{crank}, m_{piston}, and m_{conn}, if the engine is larger. A good rule of thumb is to assume that the masses scale with the product of the square of the bore and the stroke.
- Another variable is the maximum pressure in the engine. The maximum pressure p_{max} will be greater in high performance engines.

Report

A. Provide a table with specifications (analogous to Table 2) and representative plots for the engine of your choice – see the 8 items listed above. Each plot must be clearly labeled.
B. Attach a copy of the matlab script file that was used to generate the data for the plots.
C. Discuss each plot briefly. What does the plot tell you about engine performance? What do you find interesting about the plot?
D. Download the Excel spreadsheet, CarSpecifications.xls, from the class web site. Plot for all engines:
    - The brake horse power (BHP) versus the displacement volume
    - The engine speed for the peak brake horse power versus the engine size
    - The weight to power ratio versus the engine speed for maximum BHP

   Comment on the trends that you observe.

Table 3 Miscellaneous units and useful conversions

Pressure	1 bar	10^5 Newtons/meter^2
	1 atmosphere	14.7 lbs/inch^2 (psi)
	1 psi	6895 Newtons/meter^2
Volume	1 liter	1000 cubic centimeters (cc)
	1 cc	10^6 mm^3
	1 cc	10^{-3} meter^3
Mass	1 kg (weight)	2.2 pounds
Angular velocity	1 rotation per minute (RPM)	$2\pi/60$ radians per second
Power	1 Watt	1 Newton meter/second
	1 Horsepower	745.6 Watts