

# PROOF OF GOLDBACH,POLIGNAC,LEGENDRE CONJECTURE

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ABSTRACT. In this paper, I suddied about prime number's distribution

## 1. instruction

Let  $P_i$  is i-th prime number

$\bigcup_{i=1}^k \bigcup_{n=1}^{\infty} nP_i + a_i$  's longest continual number is smaller than  $2P_k$  is easily proved by mathematical induction. hence, in the  $2P_k$  continual number, it contains  $P_k$  -prime number(not divided only by prime numbers that larger than  $P_k$  ) and when  $P_k$  -prime number is smaller than  $P_{k+1}^2$  , it's a prime number

when k=2,  $\begin{matrix} 2 & \bullet & 2 & \bullet & 2 & \bullet \\ \bullet & 3 & \bullet & \bullet & 3 & \bullet \end{matrix}$  first 2,3,2 is longest part and that's lenth  $3 < 2 \cdot 3$

if when suppose k=r  $\begin{matrix} 2 & \bullet & 2 & \bullet & 2 & & 2 & \bullet \\ \bullet & 3 & \bullet & 3 & \bullet & \dots & \bullet & 3 \\ & & \vdots & & \ddots & & & \\ \bullet & \bullet & \bullet & \bullet & \bullet & P_r & \bullet & \bullet \end{matrix}$  first  $P_q - 1$  numbers are

the longest part, ( $P_q$  is a smallest prime number satisfing  $P_r < P_q < 2P_r$  ) and  $P_q - 1 < 2P_r$

when k=r+1,

only first part that is shorter than  $2P_{r+1}$  increase but other part, shape of  $n \cdot P_{r+1}$

and  $\bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + b_i$  's longest continual number is smaller than  $4P_k$

also proved similarly

when k=3,  $\begin{matrix} \bullet & 3 & \bullet & \bullet & 3 & 3 & \bullet & \bullet & 3 & \bullet \\ \bullet & \bullet & \bullet & 5 & \bullet & \bullet & 5 & \bullet & \bullet & \bullet \end{matrix}$

(skipped 2n)

2,5,2,3,2,3,2,5,2 is longest and  $9 < 4 \cdot 5$  and using same method, we get

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$\bigcup \left( \bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + b_i \right)$ 's longest continual number is smaller than  $4P_k$

**2. For any national number k, there are infinitely many pairs such that  $p - p' = 2k$**

for odd number N, and N+2 isn't a prime number, Make a set like this  

$$\begin{array}{cccc} N^2 & N^2 + 2 & \cdots & N^2 + 4N \\ N^2 + 2k & N^2 + 2k + 2 & \cdots & N^2 + 4N + 2k \end{array}$$

$$((N + 4)^2 = N^2 + 8N + 16 > N^2 + 4N + 2k)$$

from we know that there is infinitely many odd number N that N+2 isn't a prime number and from instruction

we also know  $\bigcup \left( \bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + b_i \right)$ 's longest continual number is smaller than  $4P_k$  is easily proved by mathematical induction.  
 $(N^2 + 4N + 2k + 2) - N^2 > 4P_m$ ,  $(N^2 + 4N + 2k + 2) < P_{m+1}^2$   
 $(P_m^2 \leq N < P_{m+1}^2)$

hence, for odd number N, there always exist  $P_i, P_j$  that  $N^2 < P_i < P_j < (N + 4)^2$ ,  $P_j - P_i = 2k$  ( $P_m^2 \leq N < P_{m+1}^2$ ) when N+2 isn't a prime number and k satisfying  $2N + 8 > k$  from  $(N + 4)^2 = N^2 + 8N + 16 > N^2 + 4N + 2k$

hence, For any national number k, there are infinitely many pairs such that  $p - p' = 2k$

**3. there is a prime number between  $n^2$  and  $(n + 1)^2$**

we can find  $P_m$  satisfying

$$P_m^2 < n^2 < n^2 + 1 < \cdots < (n + 1)^2 - 1 < P_{m+1}^2$$

$$\text{and } n(n^2, n^2 + 1, \cdots, (n + 1)^2 - 1) > 2P_m$$

hence, from instruction, there is a prime number between  $n^2$  and  $(n + 1)^2$  for every positive integer n

**4. Every even number greater than 2 can be expressed as a sum of two prime numbers**

from instruction we also know  $\bigcup \left( \bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + b_i \right)$ 's longest continual number is smaller than  $4P_k$  is easily proved by mathematical induction.

but, in  $\begin{array}{ccc} 2N - 3 & 2N - 5 & \cdots \\ 3 & 5 & \cdots \end{array}$  we know that  $1, 2, \cdots, 2N$ 's largest factor

is smaller than  $\sqrt{2N}$  hence from  $\bigcup \left( \bigcup_{n=1}^{\infty} 2n, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + a_i, \bigcup_{i>1}^k \bigcup_{n=1}^{\infty} nP_i + b_i \right)$ 's longest continual number is smaller than  $4P_k$ , in continual  $4 \lfloor \sqrt{2N} \rfloor$  number's that less than  $2N$  contain prime number. but, when  $4 \lfloor \sqrt{2N} \rfloor < N - 1$

, Every even number greater than 2 can be expressed as a sum of two prime numbers. but, when  $33 < N, 4 \lfloor \sqrt{2N} \rfloor < N - 1$  and we know that when  $N \leq 33$  ,Every even number greater than 2 can be expressed as a sum of two prime numbers

hence, Every even number greater than 2 can be expressed as a sum of two prime numbers

### References

no reference

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